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Saddlepoint Approximations For Option Pricing

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Abstract

In this dissertation we compute option prices, when the log of the underlying stock price follows a CGMY process, using different Saddlepoint approximation methods. The Saddlepoint approximation methods will be based on the Lugannani and Rice (1980) formula, as well as on an extension which incorporates non-Gaussian bases due to Wood, Booth and Butler (1993). We will consider Saddlepoint base distributions based on the jump diffusion models of Merton (1976) and Kou (2002). We also consider higher-order approximations for both Gaussian and non-Gaussian bases.

More specifically, we price Binary Cash or Nothing (BCON) style options and vanilla call options. We demonstrate that the results produced are accurate for certain CGMY parameters.

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Contents

1 Introduction

The purpose of this dissertation is to use the Saddlepoint approximation technique, using different base distributions, to construct the tail-end probabilities required to compute option prices for when the log of the stock price follows a Lévy process.

Firstly, we'll introduce some concepts and notation that will be used throughout this dissertation. We will consider a stock, whose price at time t is S_t, in a risk-neutral equivalent martingale measure denoted by $\mathbb{Q}$. Consider a stochastic process X_t, for time $t \geq t_0 \equiv 0$, with $X_{t_0} = 0$. The risk-free interest rate is denoted by r and the dividend yield on the stock is denoted by q - both are assumed constant. In the standard Black-Scholes model, S_t evolves as:

$$S_t = S_{t_0} \exp\left((r-q)t + X_t\right), \tag{1.1}$$

where X_t is a Brownian motion with volatility σ.

The drift of S_t under $\mathbb{Q}$ must be $r-q$. This would require $\mathbb{E}_{t_0}^{\mathbb{Q}}[\exp\left(X_t\right)] = 1$, which means that the Brownian motion in (1.1) must have a drift term equal to: $-\frac{1}{2}\sigma^2$.

The problem with this simple continuous sample path model is that it doesn't take into account the volatility smile exhibited by the implied volatilities of options, sudden jumps in stock prices, nor the asymmetric and leptokurtic (the distribution is skewed to the left, and it has heavier tails and a higher peak that that of the Gaussian distributon) features of stock prices. One way to account for all these features is to introduce Lévy processes into the modelling framework.

A Lévy process is a stochastic process with stationary and independent increments, and is continuous in probability. Brownian motion is a type of Lévy process but it is the only Lévy process with continuous paths. Therefore, all other Lévy processes are jump processes which have discontinuous sample paths and therefore they allow for large sudden moves in the underlying price process, making them more suitable processes for modelling the prices of financial assets. Additionally, jump processes can capture the effect of volatility smiles and skews which makes them attractive for derivatives pricing. All Lévy processes (except Brownian motion with no jumps) generate excess kurtosis, which in turn imply they produce curvature in the implied volatility surface. Lévy processes can also, in general, capture skewness in the risk-neutral return distributions and hence produce asymmetric implied volatility surfaces. This indicates an improvement on the Black-Scholes model. Some of the simplest types of Lévy processes consist of a Brownian motion component with one or more compound Poisson processes.

The Lévy processes that we will consider in this dissertation are the following: CGMY (2002), Merton (1976) and Kou (2002) - (the latter is also called the double exponential jump diffusion model). Not all Lévy processes have a density function in an explicit analytical form, but they all have a characteristic function, which can be used to calculate option prices.

Let us therefore introduce some more notation. Denote the characteristic function by

$\psi(u) \equiv \mathbb{E}_{t_0}^{\mathbb{Q}}[\exp(iuX_t)]$, with $\psi(u) = \exp(t\Psi(u))$, where $\Psi(u)$ is the characteristic exponent. If $\mathbb{E}_{t_0}^{\mathbb{Q}}[\exp(X_t)] \neq 1$, we can mean-correct it by replacing $\Psi(u)$ with $\Psi(u) - iu\Psi(-i)$. This is equivalent to mean-correcting X_t so that $\mathbb{E}_{t_0}^{\mathbb{Q}}[\exp(X_t)] \equiv 1$, which we assume from now on. The stock price S_t evolves as in equation (1.1), but now X_t is a mean-corrected Lévy process. Therefore, the drift of the stock price S_t is equal to the risk-free interest rate minus the dividend yield. The cumulant generating function of a distribution is the log of the distribution's moment generating function. We will denote by $k(x)$ the cumulant function of the distribution we are trying to approximate.

In order to calculate vanilla option prices, we need to obtain the probabilities that the stock price is in the money, under both the risk-neutral pricing measure and the share measure (i.e. the measure with the stock price as the numeraire). One such method to calculate these probabilities for a model where the density function is not available in an explicit analytical form, is the Saddlepoint approximation method. This technique, which has its origins in a Taylor series expansion of the Fourier inversion formula, uses the model's cumulant function, and another distribution's known probability density function (henceforth pdf) and cumulative density function (henceforth cdf) to approximate these tail-end probabilities. The distribution whose pdf and cdf functions are used in the algorithm is known as the "base" distribution. The most commonly used base distribution in calculating the Saddlepoint approximation is the standard Gaussian distribution.

Rogers and Zane (1999) use the classical Saddlepoint method (i.e. standard Gaussian base distribution) to compute option prices, by employing the Lugannini and Rice (1980) approximation. This base is also used by Xiong, Wong and Salopek (2005) for a variety of models with stochastic rates and volatilities. Carr and Madan (2008) use a slightly different approach in that they identified that a vanilla option price in the Black Scholes model can be written as a single probability with a Gaussian minus Exponential distribution. They then apply the Saddlepoint technique using the Gaussian minus Exponential base distribution to obtain this single probability, and hence the price of a vanilla call option, under more sophisticated models such as CGMY.

In this paper, we aim to go slightly further by calculating Binary Cash or Nothing (BCON) option prices and vanilla option prices for the CGMY model by using different bases. We will also consider higher-order Saddlepoint approximations to see if they can produce option values which are closer to the true option prices than those obtained by the lower-order approximations of Lugannani and Rice (1980) and Wood, Booth and Butler (1993). The rest of the dissertation is structured as follows:
Section 2 describes characteristics of Lévy processes in general, focussing mainly on results which will be used later in the dissertation;
Section 3 describes in detail the methods used to construct the Saddlepoint approximations, using as base distributions the value of a Merton (1976) jump diffusion process, or the value of a Kou (2002) jump diffusion process;
Sections 4 and 5 look at the test results computed for BCON option prices and for vanilla option prices, comparing the various methods;
Section 6 investigates the results from replacing the Gaussian Minus Exponential distribution in Carr and Madan's (2008) paper with a Merton Minus Exponential distribution;
Section 7 provides the conclusions of the project.

2 Lévy Processes

2.1 Introduction

Let $(\Omega, \mathcal{F}, \mathbb{Q})$ be a probability space, and $\{\mathcal{F}_t\}_{t_0 \leq t < \infty}$ a filtration which we assume satisfies the usual conditions, with $t_0 \equiv 0$.

Definition 2.1.1. A Lévy process X_t is a stochastic process with $X_{t_0} = 0$ with probability one, that satisfies the following conditions:

- X_t has independent increments, i.e. for any $t_0 \leq s < t$, $X_t - X_s$ is independent of $\mathcal{F}_s$.
- X_t has stationary increments, i.e. for $t_0 \leq t$ and $0 \leq s$, the distribution of $X_{t+s} - X_t$ does not depend on t.
- X_t is continuous in probability, i.e. for any $\epsilon > 0$ and $t_0 \leq s$, $\lim_{t \to s} P(|X_t - X_s| > \epsilon) = 0$.

The third condition says that jumps happen at random times, and it rules out jumps at fixed or non-random times.

A pure jump Lévy process can display either finite activity or infinite activity. In the former case, the aggregate jump arrival rate is finite, whereas in the latter case, an infinite number of jumps can occur in any finite time interval. Within the infinite activity category, the sample path of the jump process can either exhibit finite variation or infinite variation. In the former case, the aggregate absolute distance travelled by the process is finite, while in the latter case, the aggregate absolute distance travelled by the process is infinite over any finite time interval. (See Carr and Liuren (2004)).

Let $\nu(x)$ denote the Lévy density of a distribution. Essentially, this is the expected number of jumps per unit of time whose size belongs to the set x. Mathematically we have:

Proposition 2.1.2. *Let X_t be a Lévy process with triplet (γ, σ^2, ν)*

- *If $\nu(\mathbb{R}) < \infty$, then almost all paths of X_t have a finite number of jumps on every compact interval. The Lévy process has finite activity.*
- *If $\nu(\mathbb{R}) = \infty$, then almost all paths of X_t have an infinite number of jumps on every compact interval. The Lévy process has infinite activity.*

Proposition 2.1.3. *Let X_t be a Lévy process with triplet (γ, σ^2, ν)*

- *If $\sigma^2 = 0$ and $\int_{|x|\leq 1} |x|\nu(dx) < \infty$, then almost all paths of X_t have finite variation.*
- *If $\sigma^2 \neq 0$ or $\int_{|x|\leq 1} |x|\nu(dx) = \infty$, then almost all paths of X_t have infinite variation.*

The defining feature of a compound Poisson process is that there are a finite number of jumps in any finite time interval. The classical example of such a process is the compound Poisson jump diffusion process of Merton (1976). Examples of infinite activity processes are the generalized hyperbolic class of Eberlein, Keller, and Prause (1998), the variance gamma (VG) model of Madan, Carr, and Chang (1998) and its generalization to the CGMY model of Carr, Geman, Madan, and Yor (2002).

As mentioned in the Introduction, we will denote the characteristic function of a stochastic process X_t by $\psi(u) \equiv \mathbb{E}_{t_0}^{\mathbb{Q}}[\exp(iuX_t)]$. In particular, for a Lévy process, we can write the characteristic function in the form $\psi(u) \equiv \exp(t\Psi(u))$, where $\Psi(u)$ is the characteristic exponent, which is given by the Lévy Khintchine formula:

$$\Psi(u) = iu\gamma - \frac{1}{2}\sigma^2 u^2 + \int_{-\infty}^{\infty} (\exp(iux) - 1 - iux\mathbf{1}_{(|x|<1)})\nu(dx),$$

where $i = \sqrt{-1}$ and γ is the drift of the Lévy process. The term $iux\mathbf{1}_{(|x|<1)}$ is necessary, intuitively speaking, to ensure that the sum of small jumps converges, and it can in fact be omitted for Lévy processes with finite variation.

We know that the drift rate on the stock under the risk-neutral measure $\mathbb{Q}$ must be $r - q$. Therefore, we must choose the drift term γ of the Lévy process such that $\mathrm{E}_{t_0}^{\mathbb{Q}}[\exp(X_t)] = 1$, to have a model consistent with no arbitrage. Consequently we need:

$$\gamma = -\frac{1}{2}\sigma^2 - \int_{-\infty}^{\infty} (\exp(x) - 1 - x\mathbf{1}_{(|x|<1)})\nu(dx).$$

Given the characteristic exponent, we define the cumulants, c_n, via:

$$c_n = \frac{1}{i^n} \left.\frac{\partial^n \Psi}{\partial u^n}\right|_{u=0}.$$

In particular, $\mathbb{E}_{t_0}^{\mathbb{Q}}[X_1] = c_1$ and $Var_{t_0}^{\mathbb{Q}}[X_1] = c_2$. The cumulants are the derivatives of the cumulant generating function at unit time, where the cumulant generating function is the log of the moment generating function.

For processes which can, intuitively speaking, be represented as the sum of two independent processes, one producing up jumps and the other producing down jumps, we will split c_3 and c_4 into "up" and "down" components: c_3^{up}, c_3^{down}, c_4^{up}, c_4^{down} with $c_3 = c_3^{\text{up}} - |c_3^{\text{down}}|$.

2.2 The CGMY Process

The CGMY process was introduced by Carr, Geman, Madan and Yor (2002), and is also called the KoBoL process. Without a diffusion component, it is a pure jump process. For our purposes, it is convenient to consider the CGMY process as two independent processes, one producing up jumps and the other producing down jumps. Furthermore, for maximum generality, we'll consider the generalised form of the CGMY process which uses different $C_{\text{up}}, Y_{\text{up}}$ and $C_{\text{down}}, Y_{\text{down}}$ values for the up and down components.

For $Y_{\text{up}}, Y_{\text{down}} \neq 0, 1, 2$, the CGMY characteristic function for time T, is given by:

$$\begin{aligned}\psi_{CGMY}(u;\sigma_{\text{CGMY}},C_{\text{up}},C_{\text{down}},G,M,Y_{\text{up}},Y_{\text{down}}) &= \exp\Bigg(T\Bigg(C_{\text{up}}\Gamma(-Y_{\text{up}})\left[(M-iu)^{Y_{\text{up}}}-M^{Y_{\text{up}}}\right] \\ &+ C_{\text{down}}\Gamma(-Y_{\text{down}})\left[(G+iu)^{Y_{\text{down}}}-G^{Y_{\text{down}}}\right] \\ &- \frac{1}{2}\sigma^2_{\text{CGMY}}u^2\Bigg)\Bigg),\end{aligned}$$

where $C_{\text{up}}, C_{\text{down}}, G, M > 0, Y_{\text{up}}, Y_{\text{down}} < 2$, and $\Gamma(.)$ represents the gamma function. The term involving $\sigma^2_{\text{CGMY}} \geq 0$ is the variance of the Brownian component for the CGMY model. If no Brownian component is present, we set $\sigma^2_{\text{CGMY}} = 0$.

Note that the characteristic function above is **NOT** mean-corrected. The mean-corrected CGMY characteristic function for time T is:

$$\begin{aligned}\psi_{CGMY}(u;\sigma_{\text{CGMY}},C_{\text{up}},C_{\text{down}},G,M,Y_{\text{up}},Y_{\text{down}}) &= \exp\Bigg(T\Bigg(iu\gamma-\frac{1}{2}\sigma^2_{\text{CGMY}}u^2 \\ &+ C_{\text{up}}\Gamma(-Y_{\text{up}})\left[(M-iu)^{Y_{\text{up}}}-M^{Y_{\text{up}}}\right] \\ &+ C_{\text{down}}\Gamma(-Y_{\text{down}})\left[(G+iu)^{Y_{\text{down}}}-G^{Y_{\text{down}}}\right]\Bigg)\Bigg),\end{aligned}$$

where the drift γ is given by:

$$\begin{aligned}\gamma = -\frac{1}{2}\sigma^2_{\text{CGMY}} - C_{\text{up}}\Gamma(-Y_{\text{up}})\left[(M-1)^{Y_{\text{up}}}-M^{Y_{\text{up}}}\right] \\ - C_{\text{down}}\Gamma(-Y_{\text{down}})\left[(G+1)^{Y_{\text{down}}}-G^{Y_{\text{down}}}\right].\end{aligned}$$

(The condition for $Y_{\text{up}}, Y_{\text{down}} < 2$ is required to yield a valid Lévy measure). The parameters $C_{\text{up}}, C_{\text{down}}$ intuitively are measures of the overall activity; the parameters G and M control the rate of exponential decay on the left and right of the Lévy density respectively - leading to skewed distributions when they are unequal. For the CGMY model, if $M > G$, we get negative skewness which is typically what is observed in the equity options markets. The parameters $Y_{\text{up}}, Y_{\text{down}}$ determine the character of both the activity and the variation of the CGMY process. If there is no Brownian component, the CGMY process has:

$$\left\{\begin{array}{ll}\text{finite activity and finite variation} & \text{if } Y_{\text{up}}, Y_{\text{down}} < 0 \\ \text{infinite activity and finite variation} & \text{if } 0 \leq \max(Y_{\text{down}}, Y_{\text{up}}) < 1 \\ \text{infinite activity and infinite variation} & \text{if } 1 \leq \max(Y_{\text{down}}, Y_{\text{up}}) < 2\end{array}\right\}$$

If $Y_{\text{up}}, Y_{\text{down}} < 0$, then the CGMY process is a compound Poisson process.

(The characteristic function of the CGMY process has a different form if $Y_{\text{up}} = 0$ or $Y_{\text{down}} = 0$, or if $Y_{\text{up}} = 1$ or $Y_{\text{down}} = 1$. Therefore for simplicity, when discussing the CGMY prcoess, we will assume throughout this dissertation that neither Y_{up} nor Y_{down} are equal to zero or equal to one. The special case of the variance gamma process ($Y_{\text{up}}, Y_{\text{down}} = 0$) is treated separately).

The cumulants of the generalised CGMY distribution, using the method described in Section 2.1, are:

	$CGMY(\sigma_{\text{CGMY}}, C_{\text{up}}, C_{\text{down}}, G, M, Y_{\text{up}}, C_{\text{down}})$
c_1	$\gamma + C_{\text{up}} M^{Y_{\text{up}}-1}\Gamma(1 - Y_{\text{up}}) - C_{\text{down}} G^{Y_{\text{down}}-1}\Gamma(1 - Y_{\text{down}})$
c_2	$\sigma^2_{\text{CGMY}} + C_{\text{up}} M^{Y_{\text{up}}-2}\Gamma(2 - Y_{\text{up}}) + C_{\text{down}} G^{Y_{\text{down}}-2}\Gamma(2 - Y_{\text{down}})$
c_3^{up}	$C_{\text{up}} M^{Y_{\text{up}}-3}\Gamma(3 - Y_{\text{up}})$
$\lvert c_3^{\text{down}} \rvert$	$C_{\text{down}} G^{Y_{\text{down}}-3}\Gamma(3 - Y_{\text{down}})$
c_4^{up}	$C_{\text{up}} M^{Y_{\text{up}}-4}\Gamma(4 - Y_{\text{up}})$
c_4^{down}	$C_{\text{down}} G^{Y_{\text{down}}-4}\Gamma(4 - Y_{\text{down}})$

2.3 The Variance Gamma Process

The variance gamma (VG) process is a special case of the CGMY process, and can be obtained by setting $Y_{\text{up}} = Y_{\text{down}} = 0$ and $C_{\text{up}} = C_{\text{down}} = C$ in the CGMY model. Furthermore, we will assume that there is no Brownian component. The VG model has two sets of parameterization: in terms of C, G and M, and in terms of σ_{vg}, ν and θ, where the two are related by:

$$\begin{aligned} C &= 1/\nu, \\ G &= \left(\sqrt{\frac{1}{4}\theta^2\nu^2 + \frac{1}{2}\sigma^2_{\text{vg}}\nu} - \frac{1}{2}\theta\nu\right)^{-1}, \\ M &= \left(\sqrt{\frac{1}{4}\theta^2\nu^2 + \frac{1}{2}\sigma^2_{\text{vg}}\nu} + \frac{1}{2}\theta\nu\right)^{-1}. \end{aligned}$$

The characteristic function of the VG process with the $\sigma_{\text{vg}}, \nu, \theta$ parameterization for time T is given by:

$$\psi_{VG}(u; \sigma_{\text{vg}}, \nu, \theta) = \left(1 - iu\theta\nu + \frac{1}{2}\sigma^2_{\text{vg}}\nu u^2\right)^{-T/\nu},$$

or, with the C, G, M parameterization:

$$\psi_{VG}(u; C, G, M) = \left(\frac{GM}{GM + (M - G)iu + u^2}\right)^{TC}.$$

The VG process has infinite activity and finite variation. The cumulants of the VG distribution with the $\sigma_{\text{vg}}, \nu, \theta$ parameterization are:

	$VG(\sigma_{\mathrm{vg}}, \nu, \theta)$
c_1	θ
c_2	$\sigma_{\mathrm{vg}}^2 + \nu\theta^2$
c_3	$3\theta\nu\sigma_{\mathrm{vg}}^2 + 2\nu^2\theta^3$
c_4	$3\nu\sigma_{\mathrm{vg}}^4 + 12\sigma_{\mathrm{vg}}^2\nu^2\theta^2 + 6\nu^3\theta^4$

and with the C, G, M parameterization:

	$VG(C, G, M)$
c_1	$\frac{C}{M} - \frac{C}{G}$
c_2	$\frac{C}{M^2} + \frac{C}{G^2}$
c_3^{up}	$\frac{2C}{M^3}$
$\lvert c_3^{\mathrm{down}} \rvert$	$\frac{2C}{G^3}$
c_4^{up}	$\frac{6C}{M^4}$
c_4^{down}	$\frac{6C}{G^4}$

2.4 Jump Diffusion Models

A Lévy process of jump diffusion type has the following form:

$$X_t = \gamma t + \sigma W_t + \sum_{i=1}^{N_t} Y_i,$$

where W_t is the standard Brownian motion, N_t is the Poisson process counting the number of jumps, and Y_i are the independent and identically distributed jump sizes. We can trivially extend the form of a jump diffusion process to allow for multiple Poisson processes.

2.4.1 The Merton Model

The Merton jump diffusion process was introduced by Merton (1976). Although Merton only considered the case when there was a single compound Poisson process, we'll consider the case when there are two compound Poisson processes. The Merton (1976) model with two compound Poisson processes can be represented in the form:

$$X_t = \gamma t + \sigma_{\mathrm{Mert}} W_t + \sum_{i=1}^{N_t^1} Y_i^1 + \sum_{i=1}^{N_t^2} Y_i^2$$

where σ_{Mert} is the volatility of the Brownian component for the Merton model, $Y_i^1 \backsim N(\mu_1, \delta_1^2)$ and $Y_i^2 \backsim N(\mu_2, \delta_2^2)$, and where N_t^1 and N_t^2 are the Poisson processes. (We have used $N(a,b)$ to denote a Gaussian model with mean a and variance b).

The mean-corrected characteristic function for time T is given by:

$$\psi_{\text{Merton}}(u;\sigma_{\text{Mert}},\lambda_1,\mu_1,\delta_1,\lambda_2,\mu_2,\delta_2) = \exp\left[T\left(iu\gamma - \frac{1}{2}\sigma_{\text{Mert}}^2 u^2 + \lambda_1\left(\exp\left(iu\mu_1 - \frac{1}{2}\delta_1^2 u^2\right) - 1\right)\right.\right.$$
$$\left.\left. + \lambda_2\left(\exp\left(iu\mu_2 - \frac{1}{2}\delta_2^2 u^2\right) - 1\right)\right)\right],$$

where

$$\gamma = -\frac{1}{2}\sigma_{\text{Mert}}^2 - \lambda_1\left(\exp\left(\mu_1 + \frac{1}{2}\delta_1^2\right) - 1\right) - \lambda_2\left(\exp\left(\mu_2 + \frac{1}{2}\delta_2^2\right) - 1\right),$$

and $\sigma_{\text{Mert}} > 0, 0 < \lambda_1 < \infty, 0 < \lambda_2 < \infty, \delta_1 \geq 0, \delta_2 \geq 0$. In principle, the mean jump sizes μ_1 and μ_2 can take any finite, real values. However, for our purposes (see Section 3), it will be convenient to choose $\mu_1 > 0$ and $\mu_2 < 0$ (i.e. we choose them so that one mean jump size is positive and one mean jump size is negative).

The cumulants of the Merton (1976) distribution with two compound Poisson processes are:

	$Merton(\sigma_{\text{Mert}},\lambda_1,\lambda_2,\mu_1,\mu_2,\delta_1,\delta_2)$
c_1	$\gamma + \lambda_1\mu_1 + \lambda_2\mu_2$
c_2	$\sigma_{\text{Mert}}^2 + \lambda_1(\delta_1^2 + \mu_1^2) + \lambda_2(\delta_2^2 + \mu_2^2)$
c_3^{up}	$\lambda_1(3\delta_1^2\mu_1 + \mu_1^3)$
$\lvert c_3^{\text{down}}\rvert$	$\lambda_2(3\delta_2^2 + \mu_2^2)\lvert\mu_2\rvert$
c_4^{up}	$\lambda_1(3\delta_1^4 + 6\delta_1^2\mu_1^2 + \mu_1^4)$
c_4^{down}	$\lambda_2(3\delta_2^4 + 6\delta_2^2\mu_2^2 + \mu_2^4)$

The formulae for c_4 corrects a typo in Cont and Tankov (2004).

We have decomposed c_3 as $c_3^{\text{up}} - |c_3^{\text{down}}|$ and c_4 as $c_4^{\text{up}} + c_4^{\text{down}}$. This is somewhat arbitrary for the Merton (1976) process as the Lévy measure has support on the whole of the real line. However, it conforms with the intuition of the other processes we consider because we will later (see Section 3) choose $\mu_1 > 0$ and $\mu_2 < 0$.

The Merton model has a closed form pdf and cdf. The pdf $\upsilon(.)$, and cdf $\Upsilon(.)$, of the Merton model with two compound Poisson processes are:

$$\upsilon(x) = \sum_{k_1=0}^{\infty}\sum_{k_2=0}^{\infty}\frac{\exp(-\lambda_1 t)(\lambda_1 t)^{k_1}}{k_1!}\frac{\exp(-\lambda_2 t)(\lambda_2 t)^{k_2}}{k_2!}\frac{\exp\left\{-\frac{(x-\gamma t-k_1\mu_1-k_2\mu_2)^2}{2(\sigma_{\text{Mert}}^2 t+k_1\delta_1^2+k_2\delta_2^2)}\right\}}{\sqrt{2\pi(\sigma_{\text{Mert}}^2 t + k_1\delta_1^2 + k_2\delta_2^2)}}$$
$$= \sum_{k_1=0}^{\infty}\sum_{k_2=0}^{\infty} P_p(\lambda_1 t, k_1)P_p(\lambda_2 t, k_2)\phi\left(x, \gamma t + k_1\mu_1 + k_2\mu_2, \sqrt{\sigma_{\text{Mert}}^2 t + k_1\delta_1^2 + k_2\delta_2^2}\right).$$

$$\Upsilon(x) = \sum_{k_1=0}^{\infty}\sum_{k_2=0}^{\infty} P_p(\lambda_1 t, k_1)P_p(\lambda_2 t, k_2)\Phi\left(x, \gamma t + k_1\mu_1 + k_2\mu_2, \sqrt{\sigma^2_{\text{Mert}}t + k_1\delta_1^2 + k_2\delta_2^2}\right).$$

where $P_p(a, b)$ is the density (or mass function) of the Poisson distribution evaluated at a non-negative integer b, with intensity rate a; and $\phi(x, a, b)$ and $\Phi(x, a, b)$ are the density and distributions functions respectively of a Gaussian distribution evaluated at x, with mean a, standard deviation b. We will use $\phi(x)$ and $\Phi(x)$ to denote the standard Gaussian (i.e. mean 0, standard deviation 1) density and distribution functions, evaluated at x.

2.4.2 The Kou Model

Kou introduced his jump diffusion model in 2002. In this model, Y_i is a sequence of independent and identically distributed non-negative random variables such that Y_i has an asymmetric double exponential distribution. Therefore,

$$Y_i \overset{d}{=} \begin{cases} \xi^+ & \text{with probability } p \\ \xi^- & \text{with probability } 1-p \end{cases}$$

where ξ^+ and ξ^- are exponential random variables with means $\frac{1}{\eta_1}$ and $\frac{1}{\eta_2}$ respectively, and where the notation $\overset{d}{=}$ denotes "equal in distribution". We will only consider one Poisson process for this model as the Kou (2002) model already takes into account the separate up and down jumps.

The mean-corrected characteristic function for time T is given by:

$$\psi_{\text{Kou}}(u; \sigma_{\text{Kou}}, \lambda, \eta_1, \eta_2, p) = \exp\left(T\left(iu\gamma - \frac{1}{2}\sigma^2_{\text{Kou}}u^2 + iu\lambda\left(\frac{p}{\eta_1 - iu} - \frac{1-p}{\eta_2 + iu}\right)\right)\right),$$

where

$$\gamma = -\frac{1}{2}\sigma^2_{\text{Kou}} - \lambda\left(\frac{p}{\eta_1 - 1} - \frac{1-p}{\eta_2 + 1}\right),$$

and $\sigma_{\text{Kou}} > 0, \lambda > 0, \eta_1 > 0, \eta_2 > 0, 0 < p < 1$. ($\sigma_{\text{Kou}}$ is the volatility of the Brownian component for the Kou (2002) model).

The cumulants of the Kou (2002) distribution are:

	$Kou(\sigma_{\text{Kou}}, \lambda, \eta_1, \eta_2, p)$
c_1	$\gamma + \lambda(\frac{p}{\eta_1} - \frac{1-p}{\eta_2})$
c_2	$\sigma^2_{\text{Kou}} + 2\lambda(\frac{p}{\eta_1^2} + \frac{1-p}{\eta_2^2})$
c_3^{up}	$6\lambda(\frac{p}{\eta_1^3})$
$\lvert c_3^{\text{down}}\rvert$	$6\lambda(\frac{1-p}{\eta_2^3})$
c_4^{up}	$24\lambda(\frac{p}{\eta_1^4})$
c_4^{down}	$24\lambda(\frac{1-p}{\eta_2^4})$

The formulae for c_3 and c_4 correct typos in Cont and Tankov (2004).

The Kou (2002) model does not have a truly closed form pdf and cdf. However, Kou (2002) does have a semi-analytical expression for the cdf in terms of Hh functions, which is a special function used in mathematics. As a result, the pdf can be found by differentiating this probability. Recalling that the jump diffusion process is represented by the form $X_t = \gamma t + \sigma_{\text{Kou}} W_t + \sum_{i=1}^{N_t} Y_i$, the complementary cdf of the Kou (2002) model with one compound Poisson process is:

$$\begin{aligned}
\mathbb{P}(X_T \geq a) &= \frac{e^{((\sigma_{\text{Kou}}\eta_1)^2 \frac{T}{2})}}{\sigma_{\text{Kou}}\sqrt{2\pi T}} \sum_{n=1}^{\infty} \pi_n \sum_{k=1}^{n} P_{n,k} (\sigma_{\text{Kou}}\sqrt{T}\eta_1)^k I_{k-1}\left(a - \gamma T; -\eta_1, -\frac{1}{\sigma_{\text{Kou}}\sqrt{T}}, -\sigma_{\text{Kou}}\eta_1\sqrt{T}\right) \\
&+ \frac{e^{((\sigma_{\text{Kou}}\eta_2)^2 \frac{T}{2})}}{\sigma_{\text{Kou}}\sqrt{2\pi T}} \sum_{n=1}^{\infty} \pi_n \sum_{k=1}^{n} Q_{n,k} (\sigma_{\text{Kou}}\sqrt{T}\eta_2)^k I_{k-1}\left(a - \gamma T; \eta_2, \frac{1}{\sigma_{\text{Kou}}\sqrt{T}}, -\sigma_{\text{Kou}}\eta_2\sqrt{T}\right) \\
&+ \pi_0 \Phi\left(-\frac{a - \mu T}{\sigma_{\text{Kou}}\sqrt{T}}\right), \qquad (2.1)
\end{aligned}$$

where:

$$\pi_n = \mathbb{P}(N(T) = n) = \frac{e^{-\lambda T}(\lambda T)^n}{n!};$$

$$P_{n,k} = \sum_{i=k}^{n-1} \binom{n-k-1}{i-k} \binom{n}{i} \left(\frac{\eta_1}{\eta_1 + \eta_2}\right)^{i-k} \left(\frac{\eta_2}{\eta_1 + \eta_2}\right)^{n-i} p^i q^{n-i},$$

with $P_{n,n} = p^n$;

$$Q_{n,k} = \sum_{i=k}^{n-1} \binom{n-k-1}{i-k} \binom{n}{i} \left(\frac{\eta_1}{\eta_1 + \eta_2}\right)^{n-i} \left(\frac{\eta_2}{\eta_1 + \eta_2}\right)^{i-k} p^{n-i} q^i,$$

with $Q_{n,n} = q^n$.

If $\beta > 0$ and $\alpha \neq 0$, then for all $n \geq -1$:

$$I_n(c; \alpha, \beta, \delta) = -\frac{e^{\alpha c}}{\alpha} \sum_{i=0}^{n} \left(\frac{\beta}{\alpha}\right)^{n-i} Hh_i(\beta c - \delta) + \left(\frac{\beta}{\alpha}\right)^{n+1} \frac{\sqrt{2\pi}}{\beta} e^{\frac{\alpha\delta}{\beta} + \frac{\alpha^2}{2\beta^2}} \Phi\left(-\beta c + \delta + \frac{\alpha}{\beta}\right).$$

If $\beta < 0$ and $\alpha < 0$, then for all $n \geq -1$:

$$I_n(c; \alpha, \beta, \delta) = -\frac{e^{\alpha c}}{\alpha} \sum_{i=0}^{n} \left(\frac{\beta}{\alpha}\right)^{n-i} Hh_i(\beta c - \delta) - \left(\frac{\beta}{\alpha}\right)^{n+1} \frac{\sqrt{2\pi}}{\beta} e^{\frac{\alpha\delta}{\beta} + \frac{\alpha^2}{2\beta^2}} \Phi\left(\beta c - \delta - \frac{\alpha}{\beta}\right).$$

where:

$$
\begin{aligned}
Hh_n(x) &= \int_x^\infty Hh_{n-1}(y)dy, n = 0, 1, 2, ..., \\
Hh_{-1}(x) &= e^{-\frac{x^2}{2}} = \sqrt{2\pi}\phi(x), \\
Hh_0(x) &= \sqrt{2\pi}\Phi(-x).
\end{aligned}
$$

Although the equation above for $\mathbb{P}(X_T \geq a)$ is an infinite series, Kou (2002) notes that typically only 10 to 15 terms need to be calculated, depending on the required precision, as the series converges very quickly. We have run some tests (not reported) which demonstrate that for most Kou parameters, for an accuracy to 6 decimal places, between 11 to 18 terms are required to be calculated. When the Kou parameters are very small (eg. $\lambda = 0.2518, \eta_1 = 11.4425, \eta_2 = 5.7004, p = 0.4169$), less than 10 terms need to be calculated for an accuracy to 6 decimal places.

The density function can be found by analytically differentiating the probability $\mathbb{P}(X_T \geq a)$, and multiplying by minus one. We note, as an aside, that we checked our analytical formula for the density function by numerically differentiating the equation for $\mathbb{P}(X_T \geq a)$ and confirming that the results matched to a significant number of decimal places. For very extreme values of a, we did notice that the Matlab implementation of the formula for $\mathbb{P}(X_T \geq a)$ did suffer from numerical instabilities (see Appendix D for a detailed explanation).

3 Saddlepoint Method

Saddlepoint approximations are powerful tools for obtaining accurate expressions for distribution functions which are not known in closed form. Saddlepoint approximations almost always outperform other methods with respect to computational costs, though not necessarily with respect to accuracy. Suppose we have a random variable Y whose distribution function is not known in closed form. We wish to compute the probability $\mathbb{P}(Y > y)$, that the random variable exceeds some value y.

Saddlepoint approximations use a "base" distribution with known pdf and known cdf to approximate tail-end probabilities for the random variable Y. The original method of Lugannani and Rice (1980) uses the standard Gaussian distribution as the base distribution. However, any distribution with known pdf and cdf could, in principle, be used as the base. In practice, the approximation would be more accurate if we use a base distribution which, loosely speaking, resembles or behaves likes the distribution of the random variable Y that we're interested in. What "resembles" or "behaves like" means is difficult to define exactly in general, but we will provide examples later in this section. This is based on matching some low-order derivatives of the cumulant function of the random variable Y to the derivatives of the cumulant function of the base distribution, at some chosen value. When this chosen value is zero, this is equivalent to moment matching.

We are interested in evaluating probabilities under the risk-neutral measure $\mathbb{Q}$, that the stock price S_T, at time T, is greater than or less than some strike K. That is, we wish to calculate $\mathbb{P}(S_T > K)$ or $\mathbb{P}(S_T < K)$. From equation (1.1), this is the same as calculating $\mathbb{P}(X_T > \log(\frac{K}{S_{t_0}}) - (r-q)(T-t_0))$ or $\mathbb{P}(X_T < \log(\frac{K}{S_{t_0}}) - (r-q)(T-t_0))$.
If we multiply these probabilities by the discount factor $\exp(-r(T-t_0))$, we obtain the prices, at time t_0, of Binary Cash or Nothing (BCON) options (for calls and puts respectively), with maturity T.

In general (such as for the CGMY process), neither the pdf nor the cdf of X_T are known in closed form. This motivates the use of Saddlepoint approximations. The simplest Saddlepoint approximations are based on using a Gaussian distribution, which could also be viewed as the value of an approximating Brownian motion at time T. Lévy processes have independent increments and so, intuitively speaking, the Central Limit Theorem suggests that the value of the Lévy process at time T, X_T, will be better approximated by a Gaussian distribution for larger T. This suggests that using a Gaussian distribution as the base will work better for pricing options with longer maturities. Numerical examples in Rogers and Zane (1999) support this intuition.

On the other hand, Lévy processes have jumps, skewness (in general) and excess kurtosis which are features not captured by a Gaussian distribution. Therefore, in this dissertation, we will be constructing Saddlepoint approximations using the following as the base distributions: the value, at time T, of a Merton (1976) jump diffusion process with two Poisson processes and the value, at time T, of a Kou (2002) jump diffusion process. We will informally refer to these base distributions as the Merton and Kou distributions.

3.1 Saddlepoint Formula

3.1.1 Lower-Order Formula

Proposition 3.1.1. *Consider a random variable Y with cumulant function $k(t)$. Then the probability that Y exceeds some value y is approximated by Wood, Booth and Butler's (1993) lower-order Saddlepoint approximation formula for a general base distribution with cumulant function $g(t)$:*

$$\mathbb{P}(Y > y) \approx \mathbb{P}(Y_{\text{base}} > y_{\text{base}}) + h(y_{\text{base}}) \left(\frac{\sqrt{g''(s)}}{\hat{t}\sqrt{k''(\hat{t})}} - \frac{1}{s} \right), \tag{3.1}$$

where h represents the pdf of the base distribution;
$\hat{t}$ can be obtained by solving $k^{'}(\hat{t}) = y$;
s is obtained by solving: $sg^{'}(s) - g(s) = \hat{t}k^{'}(\hat{t}) - k(\hat{t})$, and then applying $\operatorname{sgn}(s) = \operatorname{sgn}(\hat{t})$;
and $y_{base} = g^{'}(s)$.

Proof. See Wood, Booth and Butler (1993). □

Corollary 3.1.2. *When the base distribution is a Gaussian distribution, equation (3.1) reduces to the Lugannani and Rice (1980) Saddlepoint approximation formula:*

$$\mathbb{P}(Y > y) \approx 1 - \Phi(s) + \phi(s) \left(\frac{1}{u} - \frac{1}{s} \right),$$

where $\hat{t}$ can be obtained by solving $k^{'}(\hat{t}) = y$;
and $u = \hat{t}\sqrt{k''(\hat{t})}$ and $s = \operatorname{sgn}(\hat{t})\sqrt{2|(y\hat{t} - k(\hat{t}))|}$.

Proof. The cumulant function of a standard Gaussian distribution and the corresponding first and second derivatives are:

$$g(x) = \frac{1}{2}x^2, \qquad g^{'}(x) = x, \qquad g^{''}(x) = 1.$$

Therefore, solving for s:

$$\begin{aligned} \hat{t}k^{'}(\hat{t}) - k(\hat{t}) &= sg^{'}(s) - g(s) \\ &= s^2 - \frac{1}{2}s^2 \\ &= \frac{1}{2}s^2. \end{aligned}$$

Therefore $s = \operatorname{sgn}(\hat{t})\sqrt{2|(y\hat{t} - k(\hat{t}))|}$, and $y_{\text{base}} = g^{'}(s) = s$.

Finally, putting this together we have:

$$\mathbb{P}(Y > y) \approx 1 - \Phi(s) + \phi(s) \left(\frac{1}{\hat{t}\sqrt{k''(\hat{t})}} - \frac{1}{s} \right),$$

as required. □

Proposition 3.1.3. *If the base distribution is a 'shift and scale' of the distribution of the random variable Y, then the results from the Saddlepoint approximation are exact.*

Proof. If the base distribution is a shift and scale of the distribution of Y, then the cumulant function and first and second derivatives of Y and the base distribution are related by:

$$k(x) = ax + g(bx), \qquad k^{'}(x) = a + bg^{'}(bx), \qquad k^{''}(x) = b^2 g^{''}(bx),$$

where a and b are constants.
Then solving for s:

$$\begin{aligned} sg^{'}(s) - g(s) &= \hat{t}k^{'}(\hat{t}) - k(\hat{t}) \\ &= \hat{t}(a + bg^{'}(b\hat{t})) - (a\hat{t} + g(b\hat{t})) \\ &= \hat{t}bg^{'}(b\hat{t}) - g(b\hat{t}). \\ sg^{'}(s) - \hat{t}bg^{'}(b\hat{t}) &= g(s) - g(b\hat{t}). \end{aligned}$$

Since $s = b\hat{t}$, we get: $sg^{'}(s) - sg^{'}(s) = g(s) - g(s) = 0$.

The third term in equation (3.1) becomes:

$$\begin{aligned} \frac{\sqrt{g^{''}(s)}}{\hat{t}\sqrt{k^{''}(\hat{t})}} - \frac{1}{s} &= \frac{\sqrt{g^{''}(b\hat{t})}}{\hat{t}\sqrt{b^2 g^{''}(b\hat{t})}} - \frac{1}{b\hat{t}} \\ &= \frac{\sqrt{g^{''}(b\hat{t})}}{b\hat{t}\sqrt{g^{''}(b\hat{t})}} - \frac{1}{b\hat{t}} \\ &= 0. \end{aligned}$$

Then the Saddlepoint formula in equation (3.1) reduces to:

$$\mathbb{P}(Y > y) = \mathbb{P}(Y_{\text{base}} > y_{\text{base}}).$$

This shows that the Saddlepoint approximation is exact in the special case of the base distribution being a 'shift and scale' of the distribution of the random variable Y. □

3.1.2 Higher-Order Formula

A higher-order Saddlepoint approximation contains more terms, and therefore, intuitively speaking, it should provide more accurate results. The notation is the same as in the lower-order formula, but we'll also need to introduce some more notation in order to simplify the formula:

$$u = \frac{\sqrt{g^{''}(s)}}{\hat{t}\sqrt{k^{''}(\hat{t})}}, \qquad \zeta^{'}_{(r)} = \frac{g^{(r)}(s)}{(g^{(2)}(s))^{\frac{r}{2}}}, \qquad \zeta_{(r)} = \frac{k^{(r)}(\hat{t})}{(k^{(2)}(\hat{t}))^{\frac{r}{2}}},$$

where $k^{(r)}(\hat{t})$ is the r^{th} derivative of the cumulant function of the distribution that we are approximating and $g^{(r)}(s)$ is the r^{th} derivative of the base distribution's cumulant function.

General Base Distribution

The higher-order Saddlepoint formula is as follows (see Taras, Cloke-Browne, Kalimtgis (2005)):

$$\mathbb{P}(Y \geq y) \approx \mathbb{P}(Y_{\text{base}} > y_{\text{base}}) + \frac{\gamma(y_{\text{base}})}{\delta}\left[\left(\frac{1}{u} - \frac{1}{s}\right) + \frac{1}{8}\left(\frac{\zeta_4}{u} - \frac{\zeta_4'}{s}\right) - \frac{5}{24}\left(\frac{(\zeta_3)^2}{u} - \frac{(\zeta_3')^2}{s}\right) \right.$$
$$\left. - \frac{1}{2\sqrt{g^{(2)}}}\left(\frac{\zeta_3}{u^2} - \frac{\zeta_3'}{s^2}\right) - \frac{1}{g^{(2)}}\left(\frac{1}{u^3} - \frac{1}{s^3}\right)\right], \quad (3.2)$$

where

$$\delta = 1 + \frac{1}{8}\zeta_4' - \frac{5}{24}(\zeta_3')^2.$$

Gaussian Base Distribution

We will use a slightly different higher-order Saddlepoint formula for the Gaussian base distribution, which is not a direct extension of the lower-order formula seen in equation (3.1). The higher-order Saddlepoint formula for a Gaussian base distribution is as follows (see Chen (2008)):

$$\mathbb{P}(Y \geq y) \approx H(-\hat{t})$$
$$+ e^{(k(\hat{t}) - y\hat{t})}\left[\operatorname{sgn}(\hat{t})\Phi\left(-|\hat{t}|\sqrt{k''(\hat{t})}\right)e^{\left(\frac{k''(\hat{t})\hat{t}^2}{2}\right)}\left(1 - \frac{\hat{t}^3 k'''(\hat{t})}{6} + \frac{\hat{t}^4 k''''(\hat{t})}{24} + \frac{\hat{t}^6 (k'''(\hat{t}))^2}{72}\right)\right.$$
$$+ \frac{1}{72\sqrt{2\pi}(k''(\hat{t}))^{\frac{5}{2}}}\left\{3k''(\hat{t})(1 - k''(\hat{t})\hat{t}^2)(\hat{t}k''''(\hat{t}) - 4k'''(\hat{t}))\right.$$
$$\left.\left. - \hat{t}(k'''(\hat{t}))^2(3 - \hat{t}^2 k''(\hat{t}) + \hat{t}^4(k''(\hat{t}))^2)\right\}\right], \quad (3.3)$$

where $H(.)$ is the Heaviside function.

3.2 Moment Matching

In this dissertation, we will use the Gaussian, Merton and Kou distributions as the base distributions. The reasons for these choices is as follows: The Gaussian distribution is the most common distribution to use as the Central Limit Theorem shows that it is a limiting distribution for, essentially, all models with independent increments and finite variance. The Merton and Kou distributions will be able to capture additional features such as skewness and kurtosis.

In order for the Merton and Kou bases to produce good results, we require, intuitively speaking, the base distributions to closely resemble the distribution that we are attempting to approximate. In principle, this can be achieved by a number of methods. One of these methods, which we will consider in this subsection, is to use moment matching.

Suppose we are given a stochastic process which has up and down jumps with a given variance c_2, and cumulants c_3^{up}, c_4^{up}, c_3^{down} and c_4^{down} for the up and down components respectively. Our aim is to find the parameters of the Merton and Kou models which have the same values of c_2, c_3^{up}, c_4^{up}, c_3^{down} and c_4^{down} as that of the model we are trying to approximate. We will never need to match the cumulant $c_1 = \mathbb{E}_{t_0}^{\mathbb{Q}}[X_1]$, since this will be determined by risk-neutral considerations.

3.2.1 The Merton Model

We have 7 parameters to estimate in the Merton model $(\sigma_{\text{Mert}}, \lambda_1, \lambda_2, \mu_1, \mu_2, \delta_1, \delta_2)$. Firstly, we note that we wish to match 5 values $(c_2, c_3^{\text{up}}$, c_4^{up}, c_3^{down} and $c_4^{\text{down}})$ with 7 parameters, so we have 2 degrees of freedom. Secondly, we wish to avoid any procedure based on a multi-dimensional least squares fit over all 7 parameters, since such procedures are often ill-conditioned and produce unstable parameter estimates. For the rest of the analysis in this section, we will assume that the cumulants c_2, c_3^{up}, c_4^{up}, c_3^{down} and c_4^{down} refer to those of the distribution we are approximating. The cumulants of the Merton (and Kou) models will be given explicitly.

We start off by setting $\delta_1 = \alpha_1\mu_1$ and $\delta_2 = \alpha_2\mu_2$, and preselecting α_1 and α_2 to be small so that the probability of the up process producing down jumps is negligible and the probability of the down process producing up jumps is negligible. In all our examples, we chose $\alpha_1 = \alpha_2 = 0.25$ because then the former probabilities are equal to $1 - \Phi^{-1}(4)$, which is certainly extremely small. Then, by dividing c_4^{up} by c_3^{up} and dividing c_4^{down} by $|c_3^{\text{down}}|$, the cumulants are now a function of only one unknown variable:

$$\frac{c_4^{\text{up}}}{c_3^{\text{up}}} = \mu_1 \left[\frac{3\alpha_1^4 + 6\alpha_1^2 + 1}{3\alpha_1^2 + 1}\right], \qquad \frac{c_4^{\text{down}}}{|c_3^{\text{down}}|} = |\mu_2| \left[\frac{3\alpha_2^4 + 6\alpha_2^2 + 1}{3\alpha_2^2 + 1}\right],$$

where we have used the cumulants of the Merton model given in the table in Section 2.4.1.

This gives us values of μ_1 and $|\mu_2|$. We set $\mu_2 = -|\mu_2|$. By matching the values of c_3^{up} and c_3^{down} with the up and down components of the third derivative of the cumulant function for the Merton model, we can obtain the values of λ_1 and λ_2. Finally, by matching the variance of both distributions, we can rearrange the equation to obtain σ^2_{Mert}, provided that the parameters are such that σ^2_{Mert} is non-negative which, in general, is not guaranteed - but which was the case for all the parameter sets considered in this dissertation. This method is flexible as, if the stochastic process whose cumulants we are trying to match has only up jumps or only down jumps, such as for example the CGYSN process of Carr and Madan (2008), we can do essentially the same procedure as above but now just fit a Merton (1976) process with one compound Poisson jump process.

3.2.2 The Kou Model

We have 5 parameters to estimate in the Kou model $(\sigma_{\text{Kou}}, \lambda, \eta_1, \eta_2, p)$.

By dividing c_3^{up} by c_4^{up} and dividing c_3^{down} by c_4^{down}, the cumulants are now a function of only one variable:

$$\frac{c_3^{\text{up}}}{c_4^{\text{up}}} = \frac{\eta_1}{4}, \qquad \frac{c_3^{\text{down}}}{c_4^{\text{down}}} = -\frac{\eta_2}{4},$$

where we have used the cumulants of the Kou model given in the table in Section 2.4.2.

This gives us values of η_1 and η_2. Then by dividing c_3^{up} by c_3^{down}, we obtain an expression in terms of the unknown p, and the known η_1 and η_2:

$$\frac{c_3^{\text{up}}}{c_3^{\text{down}}} = -\frac{p}{\eta_1^3}\frac{\eta_2^3}{(1-p)}.$$

Rearranging this equation would give us p. By matching values of c_3^{up} (or c_3^{down}) with the up (or down) components of the third derivative of the cumulant function for the Kou model, we can obtain the value of λ. Finally, by matching the variance of both distributions, we can rearrange the equation to obtain σ^2_{Kou}.

One can see that the moment matching method for the Kou model is more straightforward to fit than the Merton model, as none of Kou's parameters need to be pre-determined in order to obtain the other parameters.

3.3 Cumulant Derivative Matching at $\hat{t}$

The moment matching methodology described in Section 3.2 works reasonably well for some parameter sets (see Section 4.3), but equally it does not work as well as we would ideally like for other parameter sets. This leads us to consider an alternative, but broadly similar, methodology. Moment matching is essentially equivalent to matching the derivatives of the cumulant functions of the two different models, evaluated at zero (see Section 2.1). Observing the form of equation (3.1), we see that the cumulant function $k(t)$ is being evaluated at $\hat{t}$ (i.e. the root of $k^{'}(t) = y$). This suggests that an alternative methodology to determine the parameters of the Merton and Kou distributions would be to match the second, third and fourth derivatives of the cumulant functions, evaluating the resulting equations at $\hat{t}$. We now describe this methodology is more detail. We refer to this methodology as the cumulant derivative matching (CDM) method at $\hat{t}$.

3.3.1 Derivatives of the Cumulant Functions

Denote by $\chi(u)$ the log of the moment generating function (i.e. the cumulant generating function), for $u \in \mathbb{R}$. We denote the up and down components of the third and fourth derivatives of the cumulant generating function by $\chi^{'''}_{\text{up}}(u)$, $\chi^{'''}_{\text{down}}(u)$, $\chi^{''''}_{\text{up}}(u)$ and $\chi^{''''}_{\text{down}}(u)$. Specifically, for the CGMY model, we denote the cumulants of the third and fourth derivatives by: $\chi^{'''}_{\text{CGMY,up}}(u)$, $\chi^{'''}_{\text{CGMY,down}}(u)$, $\chi^{''''}_{\text{CGMY,up}}(u)$ and $\chi^{''''}_{\text{CGMY,down}}(u)$.

The CGMY Model

For the CGMY model, the log of the mean-corrected CGMY moment generating function is:

$$\begin{aligned}
\chi_{\text{CGMY}}(u) &= \frac{1}{2}\sigma^2_{\text{CGMY}}u^2 + C_{\text{up}}\Gamma(-Y_{\text{up}})\left[(M-u)^{Y_{\text{up}}} - M^{Y_{\text{up}}}\right] \\
&+ C_{\text{down}}\Gamma(-Y_{\text{down}})\left[(G+u)^{Y_{\text{down}}} - G^{Y_{\text{down}}}\right] \\
&- \frac{1}{2}\sigma^2_{\text{CGMY}}u - uC_{\text{up}}\Gamma(-Y_{\text{up}})\left[(M-1)^{Y_{\text{up}}} - M^{Y_{\text{up}}}\right] \\
&- uC_{\text{down}}\Gamma(-Y_{\text{down}})\left[(G+1)^{Y_{\text{down}}} - G^{Y_{\text{down}}}\right],
\end{aligned}$$

where σ_{CGMY} is the volatility of the Brownian component. The derivatives are:

$$\begin{aligned}
\chi^{'}_{\text{CGMY}}(u) = \sigma^2_{\text{CGMY}}u &+ C_{\text{up}}(M-u)^{Y_{\text{up}}-1}\Gamma(1-Y_{\text{up}}) - C_{\text{down}}(G+u)^{Y_{\text{down}}-1}\Gamma(1-Y_{\text{down}}) \\
&- \frac{1}{2}\sigma^2_{\text{CGMY}} - C_{\text{up}}\Gamma(-Y_{\text{up}})\left[(M-1)^{Y_{\text{up}}} - M^{Y_{\text{up}}}\right] \\
&\qquad - C_{\text{down}}\Gamma(-Y_{\text{down}})\left[(G+1)^{Y_{\text{down}}} - G^{Y_{\text{down}}}\right],
\end{aligned}$$

$$\begin{aligned}
\chi^{''}_{\text{CGMY}}(u) - (\chi^{'}_{\text{CGMY}}(u))^2 &= \sigma^2_{\text{CGMY}} + C_{\text{up}}(M-u)^{Y_{\text{up}}-2}\Gamma(2-Y_{\text{up}}) \\
&+ C_{\text{down}}(G+u)^{Y_{\text{down}}-2}\Gamma(2-Y_{\text{down}}), \\
\chi^{'''}_{\text{CGMY,up}}(u) &= C_{\text{up}}(M-u)^{Y_{\text{up}}-3}\Gamma(3-Y_{\text{up}}), \\
|\chi^{'''}_{\text{CGMY,down}}(u)| &= C_{\text{down}}(G+u)^{Y_{\text{down}}-3}\Gamma(3-Y_{\text{down}}), \\
\chi^{''''}_{\text{CGMY,up}}(u) &= C_{\text{up}}(M-u)^{Y_{\text{up}}-4}\Gamma(4-Y_{\text{up}}), \\
\chi^{''''}_{\text{CGMY,down}}(u) &= C_{\text{down}}(G+u)^{Y_{\text{down}}-4}\Gamma(4-Y_{\text{down}}).
\end{aligned}$$

Note that if $u = M$ or $u = -G$, in general, the derivative terms become infinite, depending on the values of Y_{up} and Y_{down}. If $u > M$ or $u < -G$, both the moment generating function and the derivatives become undefined.

The Merton Model

For the Merton (1976) model with two Poisson processes, the log of the moment generating function is:

$$\begin{aligned}
\chi_{\text{Mert}}(u) = \frac{1}{2}\sigma^2_{\text{Mert}}u^2 &+ \lambda_1\left(\exp\left(u\mu_1 + \frac{1}{2}\delta_1^2u^2\right) - 1\right) + \lambda_2\left(\exp\left(u\mu_2 + \frac{1}{2}\delta_2^2u^2\right) - 1\right) \\
&- \frac{1}{2}\sigma^2_{\text{Mert}}u - u\lambda_1\left(\exp\left(\mu_1 + \frac{1}{2}\delta_1^2\right) - 1\right) - u\lambda_2\left(\exp\left(\mu_2 + \frac{1}{2}\delta_2^2\right) - 1\right).
\end{aligned}$$

The derivatives are:

$$
\begin{aligned}
\chi'_{\mathrm{Mert}}(u) &= \sigma^2_{\mathrm{Mert}} u + \lambda_1 \hat{\mu_1}(u) A_1(u) - \lambda_1 (A_1(1) - 1) \\
&- \frac{1}{2}\sigma^2_{\mathrm{Mert}} + \lambda_2 \hat{\mu_2}(u) A_2(u) - \lambda_2 (A_2(1) - 1), \\
\chi''_{\mathrm{Mert}}(u) - (\chi'_{\mathrm{Mert}}(u))^2 &= \sigma^2_{\mathrm{Mert}} + \lambda_1(\delta_1^2 + \hat{\mu_1}(u)^2) A_1(u) + \lambda_2(\delta_2^2 + \hat{\mu_2}(u)^2) A_2(u), \\
\chi'''_{\mathrm{Mert,up}}(u) &= \lambda_1 (3\delta_1^2 \hat{\mu_1}(u) + \hat{\mu_1}(u)^3) A_1(u), \\
\chi'''_{\mathrm{Mert,down}}(u) &= \lambda_2 (3\delta_2^2 \hat{\mu_2}(u) + \hat{\mu_2}(u)^3) A_2(u), \\
\chi''''_{\mathrm{Mert,up}}(u) &= \lambda_1 (3\delta_1^4 + 6\delta_1^2 \hat{\mu_1}(u)^2 + \hat{\mu_1}(u)^4) A_1(u), \\
\chi''''_{\mathrm{Mert,down}}(u) &= \lambda_2 (3\delta_2^4 + 6\delta_2^2 \hat{\mu_2}(u)^2 + \hat{\mu_2}(u)^4) A_2(u),
\end{aligned}
$$

where $\hat{\mu_1}(u) = \mu_1 + \delta_1^2 u$, $\hat{\mu_2}(u) = \mu_2 + \delta_2^2 u$;
and $A_1(u) = \exp(\mu_1 u + \frac{1}{2}\delta_1^2 u^2)$ and $A_2(u) = \exp(\mu_2 u + \frac{1}{2}\delta_2^2 u^2)$.

The Kou Model

For the Kou (2002) model, the log of the moment generating function is:

$$
\chi_{\mathrm{Kou}}(u) = \frac{1}{2}\sigma^2_{\mathrm{Kou}} u^2 + u\lambda \left(\frac{p}{\eta_1 - u} - \frac{1-p}{\eta_2 + u} \right) - \frac{1}{2}\sigma^2_{\mathrm{Kou}} u - u\lambda \left(\frac{p}{\eta_1 - 1} - \frac{1-p}{\eta_2 + 1} \right).
$$

The derivatives are:

$$
\begin{aligned}
\chi'_{\mathrm{Kou}}(u) &= \sigma^2_{\mathrm{Kou}} u + \lambda \left(\frac{p}{\eta_1 - u} - \frac{1-p}{\eta_2 + u} \right) + u\lambda \left(\frac{p}{(\eta_1 - u)^2} + \frac{1-p}{(\eta_2 + u)^2} \right) \\
&- \frac{1}{2}\sigma^2_{\mathrm{Kou}} - \lambda \left(\frac{p}{\eta_1 - 1} - \frac{1-p}{\eta_2 + 1} \right), \\
\chi''_{\mathrm{Kou}}(u) - (\chi'_{\mathrm{Kou}}(u))^2 &= \sigma^2_{\mathrm{Kou}} + 2\lambda \left(\frac{p}{(\eta_1 - u)^2} + \frac{1-p}{(\eta_2 + u)^2} \right) + 2u\lambda \left(\frac{p}{(\eta_1 - u)^3} - \frac{1-p}{(\eta_2 + u)^3} \right), \\
\chi'''_{\mathrm{Kou,up}}(u) &= \frac{6\lambda p \eta_1}{(\eta_1 - u)^4}, \\
\chi'''_{\mathrm{Kou,down}}(u) &= -\frac{6\lambda(1-p)\eta_2}{(\eta_2 + u)^4}, \\
\chi''''_{\mathrm{Kou,up}}(u) &= \frac{24\lambda p \eta_1}{(\eta_1 - u)^5}, \\
\chi''''_{\mathrm{Kou,down}}(u) &= \frac{24\lambda(1-p)\eta_2}{(\eta_2 + u)^5}.
\end{aligned}
$$

3.3.2 Cumulative Derivative Matching at $\hat{t}$

For the rest of the analysis in this section, we will assume that the cumulants evaluated at $\hat{t}$, $\chi'''_{\mathrm{up}}(\hat{t})$, $\chi'''_{\mathrm{down}}(\hat{t})$, $\chi''''_{\mathrm{up}}(\hat{t})$ and $\chi''''_{\mathrm{down}}(\hat{t})$, refer to those of the distribution we are approximating. The cumulants of the Merton and Kou models will be given explicitly.

The Merton Model

In a similar manner as seen in Section 3.2.1, set $\delta_1 = \alpha_1 \hat{\mu_1}(\hat{t})$ and $\delta_2 = \alpha_2 \hat{\mu_2}(\hat{t})$, where α_1 and α_2 are non-negative constants. As in Section 3.2.1, we set $\alpha_1 = \alpha_2 = 0.25$.

Then as before, by dividing $\chi_{\text{up}}^{''''}(\hat{t})$ by $\chi_{\text{up}}^{'''}(\hat{t})$ and dividing $\chi_{\text{down}}^{''''}(\hat{t})$ by $|\chi_{\text{down}}^{'''}(\hat{t})|$, the cumulants are now a function of only one variable:

$$\frac{\chi_{\text{up}}^{''''}(\hat{t})}{\chi_{\text{up}}^{'''}(\hat{t})} = \hat{\mu_1}(\hat{t}) \left[\frac{3\alpha_1^4 + 6\alpha_1^2 + 1}{3\alpha_1^2 + 1} \right], \qquad \frac{\chi_{\text{down}}^{''''}(\hat{t})}{|\chi_{\text{down}}^{'''}(\hat{t})|} = |\hat{\mu_2}(\hat{t})| \left[\frac{3\alpha_2^4 + 6\alpha_2^2 + 1}{3\alpha_2^2 + 1} \right].$$

This gives us values of $\hat{\mu_1}(\hat{t})$ and $|\hat{\mu_2}(\hat{t})|$. We set $\hat{\mu_2}(\hat{t}) = -|\hat{\mu_2}(\hat{t})|$. Note that $\hat{\mu_1}(\hat{t})$ is certainly positive and $\hat{\mu_2}(\hat{t})$ is certainly negative. By using the relations: $\delta_1 = \alpha_1 \hat{\mu_1}(\hat{t})$, $\delta_2 = \alpha_1 \hat{\mu_2}(\hat{t})$, $\hat{\mu_1}(\hat{t}) = \mu_1 + \delta_1^2 \hat{t}$ and $\hat{\mu_2}(\hat{t}) = \mu_2 + \delta_2^2 \hat{t}$, we can obtain the values of $\delta_1, \delta_2, \mu_1, \mu_2$. By matching the values of $\chi_{\text{up}}^{'''}$ and $\chi_{\text{down}}^{'''}$ with the up and down components of the third derivative of the cumulant function for the Merton model, evaluated at $\hat{t}$, we can obtain the values of λ_1 and λ_2. Finally, by matching the value of $\chi_{\text{Mert}}^{''}(u) - (\chi_{\text{Mert}}^{'}(u))^2$ and the value of $\chi^{''}(u) - (\chi^{'}(u))^2$, both evaluated at $u = \hat{t}$, we will obtain σ_{Mert}^2.

The Kou Model

In a similar manner as seen in Section 3.2.2, by dividing $\chi_{\text{up}}^{'''}$ by $\chi_{\text{up}}^{''''}$ and dividing $\chi_{\text{down}}^{'''}$ by $\chi_{\text{down}}^{''''}$, the cumulants are now a function of only one variable:

$$\frac{\chi_{\text{up}}^{'''}}{\chi_{\text{up}}^{''''}} = \frac{(\eta_1 - \hat{t})}{4}, \qquad \frac{\chi_{\text{down}}^{'''}}{\chi_{\text{down}}^{''''}} = -\frac{(\eta_2 + \hat{t})}{4}.$$

Rearranging these equations gives us values of η_1 and η_2. Then by dividing $\chi_{\text{up}}^{'''}$ by $\chi_{\text{down}}^{'''}$, we obtain an expression in terms of the unknown p, and the known η_1, η_2 and $\hat{t}$. Rearranging this expression enables us to solve for p. Then by matching values of $\chi_{\text{up}}^{'''}$ (or $\chi_{\text{down}}^{'''}$) with the up (or down) components of the third derivative of the cumulant function for the Kou model, evaluated at $\hat{t}$, we can obtain the value of λ. Finally, by matching the value of $\chi_{\text{Kou}}^{''}(u) - (\chi_{\text{Kou}}^{'}(u))^2$ and the value of $\chi^{''}(u) - (\chi^{'}(u))^2$, both evaluated at $u = \hat{t}$, we will obtain σ_{Kou}^2.

3.4 Review of the Base Distributions

There are a number of reasons why we might expect accurate results from using the Kou model as the base distribution for calculating probabilities under a CGMY model. The Kou model "resembles" the CGMY distribution for the following reasons: the generalised CGMY model we're considering contains up and down jump components, and the Kou model naturally splits into up and down jump components. If we set $Y_{\text{up}} = Y_{\text{down}} = -1$ in the CGMY model, we get the Kou model as a special case. Additionally, the Lévy measure of the CGMY model monotonically declines as one moves away from the origin (in either direction). This is also true of the Lévy measure of the Kou model. It is not possible to have this latter feature in the Merton model with two compound Poisson processes.

However, the Merton pdf and cdf functions are very simple. The Kou pdf and cdf functions are much more complex and are subject to numerical instability for certain input values.

4 Test Results - Binary Cash or Nothing

Throughout this section, we will review and compare the Binary Cash or Nothing (BCON) option prices under a generalised CGMY model, with the three different Saddlepoint base distributions (Gaussian, Merton and Kou). We'll use Matlab to produce all the results. For the purposes of comparison, we need "exact" values of the option prices. We use the following formula from Bakshi and Madan (2000):

$$\mathbb{P}(S_T > K) = \frac{1}{2} + \frac{1}{\pi}\int_0^\infty Re\left(\frac{\exp(-iu\log(K))\psi_T(u)}{iu}\right)du,$$

where $\psi_T(u)$ denotes the characteristic function of the model we are interested in. The BCON option price is then obtained by multiplying the probability $\mathbb{P}(S_T > K)$ by the relevant discount factor.

In order to compute this integral, we use Matlab's built-in "quadl" function which uses a recursive adaptive Lobatto quadrature. To use this function, we first need to compute an appropriate finite upper limit for the integral. The term $\exp(-iu\log(K))$ oscillates between -1 and 1, and the real part of the term $\frac{\psi_T(u)}{iu}$ decays monotonically and rapidly (moreover, it decays exponentially, except when $Y_{\text{up}} = Y_{\text{down}} = 0$) as $u \to \infty$. Consequently, we choose the upper limit of the integral by numerically solving for the smallest value of u such that $Re\left(\frac{\psi_T(u)}{iu}\right)$ is less than some small tolerance. In all our numerical examples, we set this small tolerance equal to 10^{-11}. This provides us with an upper limit for the integral.

Matlab's quadl function then performs the integration by recursively sub-dividing the region of integration until the integral is correct to a tolerance of 10^{-11}. The results from the integral method will not literally be exact due to the specification of an upper limit, but it is clear that they will be extremely close to the true values. We can then compare the accuracy of our Saddlepoint based approximations against these values. Of course, the Saddlepoint methods will be significantly faster than computing the integral numerically.

All of the following test results have been carried out using the moment matching method described in Section 3.2 and the CDM method described in Section 3.3.

4.1 CGMY Parameters

We calculate BCON option prices using 15 sets of parameters as shown in the following table. There are at least two sets of parameters for each of the three different categories that CGMY processes fall into (finite activity; infinite activity, finite variation; and infinite activity, infinite variation). A wide range of parameters has been chosen so that we can obtain a more complete idea about how the different bases perform under varying circumstances.

Type	Name	C_{up}	G	M	Y_{up}	C_{down}	Y_{down}
infinite activity, finite variation	set1	0.4	4.14	12.3	0.21	0.4	0.21
	set2	0.874	4.667	11.876	0.51	0.925	0.58
	set3	0.874	4.667	11.876	0.51	0.874	0.51
	set4	0.3	4.6	7.8	0.9	0.25	0.88
	set5	2	5	10	0.2	2	0.2
	set6	2	5	10	0.5	2	0.5
	set7	1.8	4.5	6.7	0.95	2	0.95
	set8	0.4	4.14	12.3	0.21	1.86	0.23
	set9	0.76	4.6	9.5	0.16	1.08	0.18
infinite activity, infinite variation	set10	0.4	4.14	12.3	1.5	0.4	1.5
	set11	2	5	10	1.5	2	1.5
finite activity, finite variation	set12	0.4	5	10	-0.5	0.4	-0.5
	set13	2	5	10	-0.5	2	-0.5
	set14	2	5	8	-1.2	2	-1.2
	set15	2	5	10	-1.5	2	-1.5

CGMY Parameter Sets Used In Option Pricing

We regard values of $C_{\text{up}}, C_{\text{down}} > 1.7$ as being relatively high values, and consequently, refer to this as high C parameter sets. Furthermore, we regard values of $C_{\text{up}}, C_{\text{down}} < 0.3$ and values of $Y_{\text{up}}, Y_{\text{down}} < 0.25$ as being relatively low values, and refer to these as low C parameter sets and low Y parameter sets respectively.

4.1.1 QQ Plots

For each of the CGMY parameter sets above, QQ-plots between the Merton and Kou distributions have been carried out for $S_{t_0} = 1, K = 1, r = 0.05, q = 0.02, \sigma_{\text{CGMY}} = 0.2$ and across maturities $T = 2, 1, 0.5$ (not all plots reported). A QQ plot is a graphical method for comparing two probability distributions by plotting their quantiles against each other. If the data fall on a 45 degree line, the data have come from populations with the same distribution. Figures 1 to 3 in Appendix A contain samples of these QQ plots for $T = 1$, for two parameter sets from each category of $Y_{\text{up}}, Y_{\text{down}}$. These plots have been obtained after applying the moment matching method described in Section 3.2 to attain the appropriate parameters for the Merton and Kou distributions.

Overall, there is a relatively good fit between both distributions. This is especially so for a larger maturity, indicating that the moment matching technique is reasonable. However, for smaller maturities ($T = 0.5$), the fit between both models isn't as good. There is however a clear indication across all maturities that for parameter sets 10 and 11, the Merton and Kou bases are not linearly correlated - this can be seen in figure 2 for plots (a) and (b). We therefore might expect very different option prices for these sets of parameters from using the two different base distributions.

4.2 CGMY Results Without a Brownian Component

The first set of results are calculated for BCON call options under the CGMY model with no Brownian Motion component, $\sigma_{\text{CGMY}} = 0$ (i.e. a pure jump model). BCON option prices are calculated for each of the 15 parameter sets for maturity 1 year; initial stock price $S_{t_0} = 100$; risk-free rate $r = 0.05$; dividend yield $q = 0.02$; and strike K takes values from 60 to 140, increasing in steps of 10. Table 1 in Appendix B displays the option prices obtained. We will refer to 'out of the money' options as OTM options, 'in the money' options as ITM options and 'at the money' options as ATM options.

There are some values that the Kou model couldn't compute - for parameter sets 14 and 15, due to the Kou parameters obtained from using the moment matching method (see Appendix D for more details). Generally, the values from both the Merton and Kou bases are not as good as those obtained from the Gaussian base. However for some parameters, none of the bases have performed particularly well - these are mainly for parameter sets where $Y_{\text{up}}, Y_{\text{down}} < 0$.

4.3 CGMY Results With a Brownian Component - I

The problem we faced above where neither the Merton nor Kou models performed well may be due to using jump diffusion models to approximate a pure jump model. Dropping the diffusion component from the Merton model causes numerical implementation problems as the Merton cdf and pdf uses the diffusion component in the denominator. In order to investigate this further, we can introduce a Brownian component into the CGMY model.

We computed BCON option prices exactly as in Section 4.2, but this time, $\sigma_{\text{CGMY}} = 0.2$. The results can be seen in the following tables in Appendix B: Table 2 for $T = 1$, table 3 for $T = 2$, table 4 for $T = 0.5$. We note that in table 3, the Saddlepoint approximation has not been able to compute BCON option prices using the Kou model, for set 11. This is due to the numerical instability issues explained in Appendix D. We will refer to the results of the original moment matching method for the Merton and Kou distributions as Merton-0 and Kou-0 respectively.

4.3.1 Analysis of Results

Comparing these new results against the case where the volatility σ_{CGMY} was zero, we can see that now both the Merton and Kou base distributions produce better results. We should bear in mind that when comparing how the different bases performed in relation to one another, it is more useful to gain an impression of how each base is performing in general, as random outliers will be difficult to account for.

Best Results From Saddlepoint Approximation

There are certain parameter sets for which the different bases produce excellent results (i.e. where the results from the Saddlepoint approximation and the numerical integral match to 4 to 5 significant figures). Across all maturities, the Merton and Gaussian bases are performing well where $1 < Y_{\text{up}}, Y_{\text{down}} < 2$ and for most of the high C parameter sets where $0 < Y_{\text{up}}, Y_{\text{down}} < 1$. Additionally, for larger maturities ($T = 2$), the Merton and Gaussian bases also produce very good results for more of the parameter sets where $0 < Y_{\text{up}}, Y_{\text{down}} < 1$. The fact that the Gaussian base performs well where $1 < Y_{\text{up}}, Y_{\text{down}} < 2$ may be due to the following reason: If we examine the form of the characteristic function for the CGMY model in the limit that $Y_{\text{up}} \to 2, Y_{\text{down}} \to 2, M \to 0$ and $G \to 0$, we see that it behaves like the characteristic function of Brownian motion. Since parameter sets 10 and 11 (where $\text{Y}_{\text{up}}, Y_{\text{down}} < 2$) both use a relatively large value of 1.5, we might expect the Gaussian base to perform well here. Furthermore, a reason for the Merton base performing well might be that conditional on the number of jumps occuring, the Merton model follows a Gaussian distribution. Furthermore, for carefully chosen parameters (i.e. jump intensity rates tending to zero), the Merton model reduces to a Brownian motion. For maturities of 1 and 2 years, the Kou model performs very well for a few OTM options, and performs reasonably well across all maturities where $Y_{\text{up}}, Y_{\text{down}} < 0$ - better so than the other bases. This is almost certainly occurring because when $Y_{\text{up}}, Y_{\text{down}} < 0$, the CGMY model is a compound Poisson process, as is the Kou model. In particular, if $Y_{\text{up}} = Y_{\text{down}} = -1$, the CGMY model is the difference between two independent compound Poisson processes with exponentially distributed jumps - the same as the Kou model. The results from the Merton and Gaussian bases look promising. However, it is necessary to see how each of the base distributions perform overall.

Worst Results From Saddlepoint Approximation

In spite of improvements in the accuracy of the results for a non-zero volatility ($\sigma_{CGMY} = 0.2$) in the CGMY model, there are still some parameter sets for which some of the Saddlepoint bases are struggling to produce very good results. For example, the Gaussian base distribution, and to a lesser extent, the Merton base distribution, don't produce great values for $Y_{\text{up}}, Y_{\text{down}} < 0$, but, as mentioned above, this is where the Kou base performs reasonably well. The Kou model experiences numerical instability problems in it's complex distribution function when calculating BCON option prices for $1 < Y_{\text{up}}, Y_{\text{down}} < 2$, and also struggles to produce reliable figures for parameter sets 6 and 7 (i.e. most of the high C parameter sets where $0 < Y_{\text{up}}, Y_{\text{down}} < 1$). But again, for these parameter sets, the Merton and Gaussian base distributions produce excellent results. The numerical instability problems are caused due to the large values of the Kou parameters obtained through the moment matching method, usually when matching to CGMY parameter sets where $1 < Y_{\text{up}}, Y_{\text{down}} < 2$. For parameter set 7, $Y_{\text{up}}, Y_{\text{down}} = 0.95$, which is very close to 1, and therefore this parameter set is likely to behave like those parameter sets where $1 < Y_{\text{up}}, Y_{\text{down}} < 2$. Moreover, the reason the Kou model struggles to produce good values for high C parameter sets, is possibly due to the parameters $C_{\text{up}}, C_{\text{down}}$ controlling the "height" of the Lévy density and hence the overall intensity of jumps - therefore it provides control over the kurtosis of the distribution. The above findings suggest that a combination of the bases would produce very good results. The differences in how well the Merton and Kou bases perform where $1 < Y_{\text{up}}, Y_{\text{down}} < 2$, reflects the information we deduced from the QQ plots: for parameter sets 10 and 11 (i.e. where $1 < Y_{\text{up}}, Y_{\text{down}} < 2$), we expected the Merton and Kou bases to produce different values as the QQ plots for these sets did not line up in a linear fashion.

Overall Analysis of the Base Distributions

It would make sense to remove parameter sets 7, 10 and 11 ($1 < Y_{\text{up}}, Y_{\text{down}} < 2$) from the analysis when using a Kou base distribution, as the results here are completely wrong. Therefore, all the future analysis assumes that we will not be considering the parameter sets for which using the Kou model results in numerical instabilities.

We will now consider how each of the Saddlepoint bases perform overall. All three bases seem to be performing differently depending on the maturity being considered, so we will conduct our analysis accordingly. In order to evaluate the overall performance of each of the different base distributions, we will consider those BCON option prices where the absolute difference between option prices obtained from the Saddlepoint method and those obtained by the numerical integration method is less than 0.01, as a fairly good result. As BCON values are essentially probabilities, this implies that obtaining fairly good results from a Saddlepoint method matches to the numerical integration value to 2 significant figures.

Using the Merton base to compute BCON option prices is more effective as we increase the maturity. For a maturity of one year, the Saddlepoint method produces less reliable values for only a few strikes. However, the largest difference between the Merton Saddlepoint results and the numerical integral results being 0.024918 indicates that overall, a Merton base is very reliable for this maturity. The results are even more promising for a maturity of 2 years, where the largest difference is 0.010383, for an ITM option where $Y_{\text{up}}, Y_{\text{down}} < 0$. The Merton base performs less well for a maturity of half a year - but we note that this is where the Kou base does well.

The Kou base exhibits the opposite trends to that of a Merton base: the shorter the maturity, the better the Kou model performs. For $T = 0.5$, there are still some results for ITM options for which the Kou model doesn't produce good results. However overall, this base is performing reasonably well as the maximum difference between the option prices computed from this base and the "exact" prices computed using the numerical integral is 0.02087.

The Gaussian base performs reasonably well all round, especially for a larger maturity. This is as expected since the Central Limit Theorem suggests that the distribution of the CGMY process at time T tends to a Gaussian distribution as T becomes larger. As previously mentioned, the Gaussian base is less accurate where $\text{Y}_{\text{up}}, Y_{\text{down}} < 0$, but again, the Kou model performs well here.

Summary

We can now use this information to see which base produced the best results overall. On the whole, each of the base distributions seem to be performing quite well. The Merton and Gaussian models produce better results for an increasing maturity while the Kou model produces better results for a shortening maturity - it outperforms both the Merton and Gaussian bases across all maturities, excluding results for which the Kou model produces erroneous results (parameter sets where $1 < Y_{\text{up}}, Y_{\text{down}} < 2$, set 7 across all maturities, and for all high C parameter sets where $0 < Y_{\text{up}}, Y_{\text{down}} < 1$ for the larger maturity of 2 years). But for all these exclusions, the Merton base produces excellent results. Therefore, mixing the Merton and Kou bases in the appropriate way, would produce a base that outperforms the commonly used Gaussian base distribution for Saddlepoint approximations.

4.4 CGMY Results With a Brownian Component - II

We now repeat the tests we performed in Section 4.3 (with $\sigma_{\text{CGMY}} = 0.2$), but this time, we use the cumulative derivative matching at $\hat{t}$ (CDM) method (i.e. the methodology of obtaining Merton and Kou parameters described in Section 3.3, which matches the derivatives of the cumulant function at $\hat{t}$). We use the same option parameter sets: for maturities: 2, 1, 0.5 years; initial stock price $S_{t_0} = 100$; risk-free rate $r = 0.05$; dividend yield $q = 0.02$; strike K takes values from 60 to 140, increasing in steps of 10; and CGMY volatility $\sigma_{\text{CGMY}} = 0.2$. The results can be seen in the following tables in Appendix B: Table 2 for $T = 1$, table 3 for $T = 2$, table 4 for $T = 0.5$. We note that in table 3, the Saddlepoint approximation has not been able to compute BCON option prices using the Kou model, for set 11. This is due to the numerical instability issues explained in Appendix D.

We will refer to the results of the CDM method for the Merton and Kou distributions as Merton-$\hat{t}$ and Kou-$\hat{t}$ respectively.

4.4.1 Comparison Between Parameter Attaining Methods

We will now explore for which strikes, and for which parameter sets the CDM method has brought about improvements in the option prices calculated using the Saddlepoint approximation technique. We'll investigate those values for which there is a large enough difference between the option prices calculated using the original moment matching method, and the CDM method. It is necessary to define what qualifies as a "large enough difference" between the results obtained from using both parameter attaining methods. We'll consider those results for which the difference is greater than 0.005 as a large enough difference. This is a tight restriction, but it will enable us to determine whether the CDM method really improves upon the original moment matching method. It would be preferable to use mainly one of the methods (i.e. moment matching method or CDM method) for the bulk of the parameter sets and range of strikes rather than single out certain strikes for the different parameter sets where each method did well. Therefore, we shall conduct our analysis based on general results, where there is an obvious pattern forming.

Two-Year Maturity

For a maturity of 2 years, there were only three differences between both the methods for the Kou model. The Merton model experienced differences for ITM options where $Y_{\text{up}}, Y_{\text{down}} < 0$ and for parameter sets 1, 3 and 9 where $0 < Y_{\text{up}}, Y_{\text{down}} < 1$. The CDM method produced better results for most of these differences, except for deep ITM options.

One-Year Maturity

For a maturity of 1 year, the Kou model only experiences a few differences between the original moment matching method and the CDM method. These were for ITM options for parameter set 1, where the CDM method produced better results. In the Merton model, the differences between the original moment matching method and the CDM method mirrored the findings for $T = 2$. However, for $T = 1$, the original moment matching method produced better results using a Merton base distribution for parameter sets where $Y_{\text{up}}, Y_{\text{down}} < 0$, indicating that for a smaller maturities the CDM method has not brought about major improvements compared to the original moment matching method.

Half-Year Maturity

The results for the CDM method were varied for the smallest maturity of half a year. The Kou model experienced large differences between the results from the two methods for ITM options for all parameter sets where $0 < Y_{\text{up}}, Y_{\text{down}} < 1$, excluding the high C parameter sets, and for ITM options for parameter set 12. Except for parameter set 12, the majority of these results produced more accurate BCON option prices using the CDM method. For the Merton model, there were mainly differences for ITM options where $Y_{\text{up}}, Y_{\text{down}} < 0$, and for ITM options for nearly all the parameter sets where $0 < Y_{\text{up}}, Y_{\text{down}} < 1$. For the Merton model, using the moment matching method produced better results for ITM options for parameter sets 3, 4, 9, 13 and 14.

Summary

Overall, these findings suggest that for the Merton model, the original moment matching method produces better results for some ITM BCON options, and overall for the Kou model, the CDM method produces better results.

4.4.2 Analysis of Results

For all maturities, the CDM method for the Merton and Kou base distributions produce the same trends as the original moment matching method for each of the maturities: The Merton model produces more accurate figures for increasing maturities, and conversely, using a Kou model produces more accurate figures for decreasing maturities. Furthermore, when comparing the overall performance between the three different base distributions using the CDM method, the results are similar to those obtained using for the original moment matching method.

Overall, the CDM method has not brought about major improvements in the accuracy of BCON option prices compared to the moment matching method. We will investigate in Section 5 whether the CDM method is able to give better results compared to the moment matching method when used to calculate vanilla option prices.

4.5 Higher-Order Approximation

We will now conduct tests which use the higher-order Saddlepoint formulae: equation (3.2) for the Merton and Kou base distributions, and equation (3.3) for the Gaussian base distribution.

The same tests that were run for BCON option pricing using the lower-order Saddlepoint approximation have been run again, but now we only consider using the original moment matching method as the CDM method didn't prove to be a great improvement. The results from the higher-order approximation can be seen in the following tables in Appendix B: Table 2 for $T = 1$, table 3 for $T = 2$, table 4 for $T = 0.5$.

4.5.1 Analysis of Results

We will now analyse the results for which the higher-order approximation produced better results than those obtained using the lower-order approximation. It will be useful to investigate for which parameter sets and strikes the higher-order results differed from the lower-order results by a large difference. This will enable us to identify where the higher-order approximation is doing particularly well. Again, as seen in the comparison between the original moment matching method and CDM method, we will consider a large difference to be 0.005.

Two-Year Maturity

For $T = 2$, the higher-order approximation for the Merton base distribution outperforms the lower-order approximation mostly for ITM options where $0 < Y_{\text{up}}, Y_{\text{down}} < 1$. For the Kou and Gaussian models, the higher-order approximation does well for some of the parameter sets where $0 < Y_{\text{up}}, Y_{\text{down}} < 1$. However, if we just focus on those results where the BCON option prices obtained from implementing the lower-order formula differ from the prices obtained from implementing the higher-order formula by more than 0.005, we find that none of these include the results for which the higher-order base performed very well.

One-Year Maturity

For $T = 1$, the higher-order approximation using the Merton base distribution does well across the range of strikes for most parameter sets where $Y_{\text{up}}, Y_{\text{down}} > 0$. The results from the Kou base distribution outperform the results from the lower-order approximation for deep ITM options where $Y_{\text{up}}, Y_{\text{down}} < 1$. The Gaussian base distribution outperforms the lower-order approximation across all parameters sets. However, there are no parameter sets for which these results differ from the lower-order approximation by more than 0.005. This indicates that the higher-order approximation hasn't yet brought about a major improvements in the results.

Half-Year Maturity

Finally, for $T = 0.5$, using the Merton base distribution produces results that outperform the results from the lower-order approximation across the range of strikes for most parameter sets where $Y_{\text{up}}, Y_{\text{down}} > 0$. The higher-order approximation for the Kou model hasn't performed as well as for the larger maturites: there are only a few very good results produced using the higher-order approximation - none of which form a logical pattern. Finally, for the Gaussian base distribution, the higher-order Saddlepoint approximation outperforms the lower-order approximation across all parameters sets. However, for the Merton and Kou models, the results from the higher-order approximation do not differ from the original results by a large amount, and for the Gaussian model, the higher-order approximation only makes a definitive impact on parameter sets 1 and 12.

Summary

We find that although the higher-order approximation performs well, it doesn't outperform the lower-order approximation by a large amount for any of the three base distributions. This indicates that the extra computational cost involved in calculating the higher-order formula isn't weighing out the benefit of a slightly more accurate BCON option price. This may be because our findings in this section display that using the three base distributions would already provide accurate results. We will investigate in Section 5 whether the higher-order approximation is able to give better results for vanilla options than the lower-order approximation.

5 Test Results - Vanilla Options

In the previous section, we considered the prices of BCON options. In this section, we will consider the prices of vanilla (standard European) options under a generalised CGMY model using the three different Saddlepoint base distributions. The exact option prices that will be used to compare the Saddlepoint results against in this section will be Carr and Madan's FFT formula, details of which can be found in Carr and Madan (1999). We will also calculate option prices using the numerical integral that was introduced in Section 4 - implemented into a "Black-Scholes-style" formula (see Section 5.1). Both techniques of obtaining the Merton and Kou parameters (the original moment matching and CDM methods) described in Section 3 will be applied.

5.1 Saddlepoint Approximations Under the Share Measure

In order to produce vanilla call option prices, we'll need to compute two probabilities. This results in a "Black-Scholes-style" option pricing formula. The price of a vanilla call option at time t_0 is:

$$\begin{aligned} C_{t_0} &= e^{-r(T-t_0)}\mathbb{E}^{\mathbb{Q}}[(S_T - K)\mathbf{1}_{\{S_T>K\}}] \\ &= e^{-r(T-t_0)}\mathbb{E}^{\mathbb{Q}}[S_T\mathbf{1}_{\{S_T>K\}}] - e^{-r(T-t_0)}\mathbb{E}^{\mathbb{Q}}[K\mathbf{1}_{\{S_T>K\}}] \\ &= e^{-q(T-t_0)}\mathbb{P}^{\mathbb{R}}[S_T > K] - Ke^{-r(T-t_0)}\mathbb{P}^{\mathbb{Q}}[S_T > K]. \end{aligned}$$

We have calculated the second probability $\mathbb{P}^{\mathbb{Q}}[S_T > K]$ in the last equation above, in Section 4. It is the price of a BCON option, multiplied by a deterministic factor, K. However, we still need to calculate the first probability, $\mathbb{P}^{\mathbb{R}}[S_T > K]$. This is the probability that S_T exceeds K under the share measure $\mathbb{R}$, where the stock is taken as the numeraire.

Rogers and Zane (1999) show that the cumulant function $k^{\mathbb{R}}(z)$ under the share measure $\mathbb{R}$ is related to the cumulant function under the risk-neutral pricing measure $\mathbb{Q}$ as follows:

$$k^{\mathbb{R}}(z) = k^{\mathbb{Q}}(z+1) - k^{\mathbb{Q}}(1).$$

By replacing the cumulant function under the risk-neutral pricing measure $k^{\mathbb{Q}}(z)$ with $k^{\mathbb{R}}(z)$, in the probabilities calculated in section 4, we are able to compute $\mathbb{P}^{\mathbb{R}}[S_T > K]$ and hence vanilla option prices under the CGMY model using the Saddlepoint technique.

5.2 Vanilla Option Pricing Results

We calculated option prices for the same option parameters seen in Section 4: for maturities: 2, 1, 0.5 years; initial stock price $S_{t_0} = 100$; risk-free rate $r = 0.05$; dividend yield $q = 0.02$; strike K takes values from 60 to 140, increasing in steps of 10; and CGMY volatility $\sigma_{\text{CGMY}} = 0.2$. The results can be seen in the following tables in Appendix B: Table 5 for $T = 1$, table 6 for $T = 2$, table 7 for $T = 0.5$. We note that in table 6, the Saddlepoint approximation has not been able to compute vanilla option prices using the Kou model, for parameter set 11. This is due to the numerical instability issues explained in Appendix D.

5.2.1 Comparison Between Parameter Attaining Methods

It seems that the CDM method is producing better results than the original moment matching method, so it's worth comparing the results between them to see if we can eliminate the use of the original method of attaining Merton and Kou parameter values for vanilla option pricing purposes. It is necessary to define what qualifies as a "large enough difference" between the results for both moment matching techniques, in order to isolate those values to see if the CDM method is performing better or not, in comparison to the original moment matching method. As the FFT method has calculated a few option prices with values less than 1 (for deep OTM options with a very small maturity - most of them are for parameter sets where $Y_{\text{up}}, Y_{\text{down}} < 0$), for these options, we'll consider a large difference to be greater than 0.005, and for all other cases, we'll consider a value of 0.05 to be a large difference. (This implies we're investigating the performance of the CDM method specifically for those results which do not match to 2-3 significant figures to results obtained using the original moment matching method). Again, as the three bases produce different results for varying maturities we'll consider the results for each maturity separately.

Two-Year Maturity

At $T = 2$, the Kou model experiences most of it's differences for half of the parameter sets where $0 < Y_{\text{up}}, Y_{\text{down}} < 1$. Excluding parameter set 4, the CDM method has produced the more accurate results. For the Merton model, the majority of the large differences fall within the parameter sets where $Y_{\text{up}}, Y_{\text{down}} < 0$, and for OTM options where $0 < Y_{\text{up}}, Y_{\text{down}} < 1$. Nearly all of these parameter sets produced better results using the CDM method.

One-Year Maturity

For a maturity of 1 year, the Kou model experienced differences in the option prices between the two methods (original moment matching and CDM methods) mostly for OTM options, where $0 < Y_{\text{up}}, Y_{\text{down}} < 1$ and for parameter sets 13 and 15. For each of these option prices, again, the CDM method produced better results. These results include the high C parameter sets where $0 < Y_{\text{up}}, Y_{\text{down}} < 1$. We emphasize that in the BCON case, this has usually been the area for which the Kou model doesn't produce good results, implying that the CDM method has brought about a major improvement. For the Merton base distribution, most of the largest differences occur for the parameter sets where $Y_{\text{up}}, Y_{\text{down}} < 1$, excluding set 7. (Therefore, there isn't a large difference in the results for where the Merton model performed exceptionally well in pricing BCON options: for parameter sets where $1 < Y_{\text{up}}, Y_{\text{down}} < 2$). Again, the option prices produced using the CDM method provided the better results overall. So for pricing vanilla options with a maturity of one year, using the CDM method certainly provides an improvement in the results.

Half-Year Maturity

Finally, for a maturity of half a year, (removing the erroneous values from sets 7 and 10 from the Kou model), there are differences for OTM options where $0 < Y_{\text{up}}, Y_{\text{down}} < 1$, most of which produced more accurate option prices using the CDM method. Furthermore, for the Merton model, there are differences for all sets (excluding parameter set 9) where $Y_{\text{up}}, Y_{\text{down}} < 1$. Except for a few deep ITM and deep OTM options, most of these parameter sets produced better results using the CDM method. There were also large differences for the Merton and Kou models for all the deep OTM options where the FFT option price was less than 1. The results for these were mixed, and there isn't a logical pattern forming for which option prices were in favour of the CDM method. However, there are only 10 of these options.

Summary

We imposed a tight restriction on what we considered a large difference, and although there were many different combinations, the above analysis indicates that the CDM method has brought about an improvement in using the Saddlepoint approximation to price vanilla options. Therefore, for the rest of the analysis, we'll only consider values obtained from applying the CDM method.

5.2.2 Analysis of Results

Best Results From Saddlepoint Approximation

As seen in the BCON case, there are certain parameter sets for which the different base distributions produce excellent results (where the results from the Saddlepoint approximation and the numerical integration match to 3 or 4 significant figures). There are similarities between the results for the BCON option prices and the vanilla option prices regarding for which set of parameters each of the three base distributions performs particularly well. (i.e. Across all maturities, the Merton and Gaussian bases perform well where $1 < Y_{\text{up}}, Y_{\text{down}} < 2$ and for most of the high C parameter sets where $0 < Y_{\text{up}}, Y_{\text{down}} < 1$. For larger maturities these two base distributions also produce very good results for more of the parameter sets where $0 < Y_{\text{up}}, Y_{\text{down}} < 1$; the Kou base distribution produces the best results for the parameter sets where $\text{Y}_{\text{up}}, Y_{\text{down}} < 0$). Additionally, for a maturity of 2 years, the Kou base distribution performs well for deep ITM and deep OTM options for parameter sets 1, 8 and 9 (low Y parameter sets where $0 < Y_{\text{up}}, Y_{\text{down}} < 1$).The analysis conducted in Section 4 into why certain base distributions perform well for certain parameter sets still holds for the vanilla option pricing case. Except for set 11 for $T = 0.5$ where none of the base distributions produce good results, using the Saddlepoint approximation produces trends that mirror the trends observed for the BCON case.

Worst Results From Saddlepoint Approximation

Again, as seen above, there are similarities between the results for the BCON option prices and the vanilla option prices regarding for which set of parameters each of the three base distributions doesn't perform so well. (i.e. For the Kou model, this is for parameter sets where $1 < Y_{\text{up}}, Y_{\text{down}} < 2$, and for high C parameter sets where $0 < Y_{\text{up}}, Y_{\text{down}} < 1$; the Gaussian model, and to a lesser extent the Merton model, struggle to produce very good values for some of the parameter sets where $Y_{\text{up}}, Y_{\text{down}} < 0$). Again, the analysis conducted in Section 4 into why certain base distributions do not perform so well for certain parameter sets still holds for the vanilla option pricing case.

Although the Kou model doesn't produce exceptional results for various parameter sets, it didn't for the BCON results either, but overall it still performed well and in particular, for smaller maturities, it outperformed the other two base distributions.

Overall Analysis of the Base Distributions

In order to evaluate the overall performance of the Saddlepoint approximation using the different base distributions to calculate vanilla option prices, we need to quantify what we consider a large enough difference between the prices obtained from using the Saddlepoint method, and the option prices computed using the FFT method. For BCON results, we regarded a difference of less than 0.01 to be fairly good (i.e. 2 significant figures), as we were investigating errors for probabilities. Therefore, in order to evaluate the performance of the Saddlepoint method to calculate vanilla option prices, for FFT values greater than 1, we'll regard a difference of less than 0.2 as a fairly good result, and for those FFT values which are less than 1 (i.e. Deep OTM options for a small maturity - most of them are for parameter sets where $Y_{\text{up}}, Y_{\text{down}} < 0$), we'll regard a difference between the Saddlepoint results and the FFT results of less than 0.02 as a fairly good result. Additionally, as mentioned in the analysis conducted for the BCON case, it is only useful to consider the performance of parameter sets and varying strikes as a whole to determine how a base is performing, as isolating single outliers is inefficient.

When $T = 2$, both the Merton and Gaussian base distributions have proved they are performing well, except for ITM options for CGMY parameter set 11. It was mentioned earlier that none of the base distributions were performing particularly well for this parameter set. Again, disregarding the sets for which the Kou model produces completely unreliable results, the Kou model's performance is not bad overall for a larger maturity, which reflects what was inferred from the BCON option analysis. However, there are now additional problems for sets where $0 < Y_{\text{up}}, Y_{\text{down}} < 1$.

For a maturity of 1 year, the Merton model proves itself to be a reliable base, as only a few outlying results differ from the FFT results by more than 0.2. The results from the Kou model are also promising: Kou only underperforms for high C parameter sets where $0 < Y_{\text{up}}, Y_{\text{down}} < 1$. However, in this case, the Kou model does produce better results for more deep OTM options. The Gaussian base generally performs well, except at high C combinations for $Y_{\text{up}}, Y_{\text{down}} < 0$. However, we've seen for the BCON results that Gaussian generally underperforms where $Y_{\text{up}}, Y_{\text{down}} < 0$, which is where the Kou model performs rather well.

Finally, for a maturity of half a year, we'll need to consider those option prices computed using the FFT method which are less than 1 separately to those prices which are greater than 1. However, where there are erroneous results for option prices less than 1, even the FFT and numerical integral values struggles to produce similar prices that agree to 2 decimal places. Therefore it would be unwise to expect very good results from the Saddlepoint approximation method. For FFT option values less than 1, both the Merton and Gaussian base distributions experience problems in computing accurate values for most of the option prices. The Kou base however, has performed very well here, it's main problems being with deep OTM options for set 12. Now we'll investigate those FFT values which are greater than 1. As seen in the BCON option case, the Merton and Gaussian models perform less well for a shorter maturity: The Merton model experiences problems calculating the prices of ITM options for sets 1 and 14. For a Gaussian base, around the ATM point (i.e. for strikes of 90, 100, 110), there are mainly problems with the option prices produced where $Y_{\text{up}}, Y_{\text{down}} < 0$. The results for the Kou base distribution are better than using a Merton base (once the erroneous results for sets 7, 10 and 11 are removed). There are however problems for ATM options for parameter sets 8 and 9.

Summary

As we would expect, pricing vanilla call options using the three Saddlepoint base distributions produces similar trends to that observed from pricing BCON style options. The Merton and Gaussian bases both perform better for a larger maturity, and conversely the Merton distribution performs better for a shorter maturity. The parameter sets for which each base distribution produces excellent results when pricing BCON options are the same for when pricing vanilla options.

5.3 Higher-Order Approximation

Although the BCON results from the higher-order Saddlepoint approximation method (see equations (3.2) and (3.3)) didn't outperform the lower-order Saddlepoint results by a relatively large amount, it is still worth exploring the effect a higher-order approximation has on pricing vanilla options, as the CDM method of attaining Merton and Kou parameter sets was more effective when calculating vanilla option prices compared to calculating BCON option prices.

The same tests as that carried out for the lower-order approximation to price vanilla call options was performed for the higher-order approximation. We will only be performing these tests using the CDM method of attaining the Merton and Kou parameters. The results can be seen in the following tables in Appendix B: Table 5 for $T = 1$, table 6 for $T = 2$, table 7 for $T = 0.5$. (We note that in table 6, the Saddlepoint approximation has not been able to compute vanilla option prices using the Kou model, for set 11. This is due to the numerical instability issues explained in Appendix D).

5.3.1 Analysis of Results

As seen for the comparison between the moment matching and CDM methods, we will consider a difference of 0.05 between the lower-order and higher-order results as a large difference. Again the results look very promising as there is a distinct pattern forming for when the results from the higher-order approximation outperform those from the lower-order approximation. However, across these results, none outperform the lower-order approximation results by a large amount.

Two-Year Maturity

For $T = 2$, as seen in the BCON case, the higher-order approximation using the Merton base distribution does well for parameter sets where $0 < Y_{\text{up}}, Y_{\text{down}} < 1$ (excluding parameter set 1) and parameter sets 10 and 13. For the Kou model, excluding parameter set 1 as for the Merton model, the higher-order approximation produces results that outperform the lower-order results for deep OTM options where $0 < Y_{\text{up}}, Y_{\text{down}} < 1$. Finally, for the Gaussian base distribution, the higher-order approximation produces results that outperform those produced using the lower-order approximation for OTM options across most of the parameter sets. Except at parameter set 13 for the Merton model, the higher-order results do not differ from the lower-order results by more than 0.05, indicating that the higher-order approximation hasn't yet brought about a significant improvement in the results.

One-Year Maturity

Using the Merton base distribution, the higher-order approximation produces better results than the lower-order approximation across the varying strikes for parameter sets where $Y_{\text{up}}, Y_{\text{down}} > 0$. The exception is for parameter set 1. For the Kou model, excluding parameter set 1 again as for the Merton model, the higher-order approximation produces results that outperform the lower-order results for deep OTM options where $0 < Y_{\text{up}}, Y_{\text{down}} < 1$. Finally, using the Gaussian base distribution, the higher-order approximation produces results that outperform those produced using the lower-order approximation for OTM options for parameter sets where $Y_{\text{up}}, Y_{\text{down}} < 0$. However, none of these results differ from those produced by implementing the lower-order Saddlepoint formula by more than 0.05. The results so far indicate that there doesn't seem to be strong evidence to support the use of the more computationally expensive higher-order Saddlepoint approximation.

Half-Year Maturity

Finally, at the shortest maturity, for the Merton model, excluding sets 1, 4, and 9, the higher-order approximation outperforms the lower-order approximation mostly for OTM options where $0 < Y_{\text{up}}, Y_{\text{down}} < 1$, and for all parameter sets where $Y_{\text{up}}, Y_{\text{down}} > 1$. The Kou model experiences good results from the higher-order approximation for deep OTM options for parameter sets 2, 3, and 8. For the Gaussian base distribution, the higher-order approximation produces results that outperform those produced using the lower-order approximation for OTM options for parameter sets where $Y_{\text{up}}, Y_{\text{down}} > 0$. However, again, the results from the lower-order approximation only differ from the higher-order approximation results by less than 0.05.

Summary

In summary, as seen for the BCON option case, even though the higher-order Saddlepoint method produces excellent results, the results don't outperform the lower-order results by a significant amount. One possible reason for this could be that the lower-order approximation already produced fairly accurate vanilla option prices across all bases, and it would be difficult to produce even better results using the Saddlepoint method for which we can't control the accuracy, as we can for the numerical integration we have seen in Section 4.

5.4 Comparison against Published Results

As mentioned in the Introduction, Saddlepoint techniques with various bases have been used to calculate option prices in several published papers. In this section, we will reproduce the same values to check the accuracy of our method using the three different base distributions.

5.4.1 Comparison Against Carr and Madan (2008)

Carr and Madan identified that the price of a call option in the Black-Scholes model could be written as a single probability with a Gaussian Minus Exponential distribution. They then used the Saddlepoint method to calculate option prices under the CGMY model using the Gaussian minus Exponential base distribution.
The CGMY parameters used are: $C_{\text{up}}, C_{\text{down}} = 2, M = 5, G = 10, Y_{\text{up}}, Y_{\text{down}} = 0.5$.
The option parameters used are: maturity $T = 0.5$; initial stock price $S_{t_0} = 100$; risk-free rate $r = 0.03$; dividend yield $q = 0$; strike K takes values from 10 to 200, increasing in steps of 10; and CGMY volatility $\sigma_{\text{CGMY}} = 0$.

Table 8 in Appendix B contains the Saddlepoint approximation results from Carr and Madan (2008), our Saddlepoint results using the Merton, Kou and Gaussian bases distributions, and the correct option prices given by the FFT method to compare against.

For strikes from 20 to 140, the Kou base distribution performs better than the base distribution Carr and Madan (2008) use. For strikes of 140 to 200, the Gaussian and Merton bases perform quite well, but not as well as Carr and Madan's (2008) base. Carr and Madan specifically aim to price deep out the money options. We have excellent results here as the range of strikes for which the option prices obtained from the Kou model outperform the prices in Carr and Madan (2008) are for the more commonly traded strikes.

5.4.2 Comparison Against Sepp (2004)

Sepp (2004) uses a numerical inversion of the Laplace transform technique to price vanilla call options using the Kou model for a range of stock prices and Poisson process intensity rates.
The Kou parameters used are: $\eta_1 = \eta_2 = 10, p = 0.5, \lambda = 0, 3, 5$.
The option parameters used are: maturity $T = 1$; initial stock prices $S_{t_0} = 90, 100, 110$; risk-free rate $r = 0.05$; dividend yield $q = 0.02$; strikes $K = 100$; and Kou volatility $\sigma_{\text{Kou}} = 0.2$.

By comparing the CGMY and Kou characteristic functions, given the Kou parameters above, we can read off what values the CGMY model should take. In particular,
$M = G = 10, Y_{\text{down}} = Y_{\text{up}} = -1$. The results from Sepp (2004) and our lower-order Saddlepoint results are in the following tables in Appendix B: Table 9 for $\lambda = 0$, table 10 for $\lambda = 3$ and table 11 for $\lambda = 5$. These tables also contain "exact" option prices from the numerical integration method which is to be used as a comparison value. The Merton base distribution results are not available for the intensity of $\lambda = 0$.

The results using the Kou base distribution are very accurate (to 10 significant figures), for a small intensity rate of $\lambda = 0$ as they match with the values from Sepp (2004), and the comparison values from the numerical integration. As the intensity rate increases, the Kou model still performs well. The Gaussian base also performs well, but as would be expected, it performs slightly less well as the intensity rate increases.

It is clear that all three Saddlepoint methods produce very accurate results. Of course, from Proposition 3.1.3, we know that using a Kou base should produce essentially exact results which our numerical results confirm. However, we also see that the Merton and Gaussian bases perform very well.

5.4.3 Comparison Against Rogers and Zane (1999)

Rogers and Zane (1999) use a Gaussian base Saddlepoint approximation to compute put option prices for three different models. One of the models used is the variance gamma (VG) model, a special case of the CGMY model.
The corresponding CGMY parameterisation used in these tests are: $C_{\text{up}}, C_{\text{down}} = 1$, $G = M = 7.071068, Y_{\text{up}}, Y_{\text{down}} = 0$.
The option parameters used are: maturity $T = 0.25, 0.5, 0.75, 1$ years; initial stock price $S_{t_0} = 1$; risk-free rate $r = 0.05$; dividend yield $q = 0$; strikes $K = \exp(-0.05), \exp(0), \exp(0.05)$; and CGMY volatility $\sigma_{\text{CGMY}} = 0$.

Table 12 in Appendix B contains the values Rogers and Zane (1999) obtained from their Saddlepoint approximation, the results from our Saddlepoint approximation using the Merton, Kou and Gaussian base distributions, and option prices obtained from the FFT method to use as a comparison. Appendix C contains graphs related to the option prices computed, which will be described in more detail later.

The Kou base distribution has performed better overall, as it consistently outperformed Rogers and Zane's (1999) Gaussian Saddlepoint results, except at a maturity of 5 years. Also, it has performed better than our Gaussian base, except at maturities of 2 and 5 years. These findings fall in line with our previous analysis that the Kou model doesn't produce accurate values for larger maturities. However on this occasion, we have not experienced the same numerical instability issues for large maturities. There is a problem with the option price obtained from all of our Saddlepoint bases when $K = \exp(0.05)$ for a 2 year maturity - we will discuss this in more detail later.

Our Gaussian base Saddlepoint has also performed well. Some of the results obtained are different to the Gaussian Saddlepoint results from Rogers and Zane (1999), but for those combinations of the maturity and strike where our Gaussian base distribution hasn't performed as well, the values are mostly within 10e-4 of the values from Rogers and Zane (1999).

The Merton base gives varied results. It produces good results across all maturities, but only outperforms the results from Rogers and Zane (1999) for random combinations of the maturities and the strikes. Since there is no exact combination for where the Merton base definitely outperforms the published results, a combination of using a Kou base for the smaller maturities, and a Gaussian base for the maturity of 5 years, would provide a good base to rival the results from Rogers and Zane (1999).

All three Saddlepoint bases return strange values when $K = \exp(0.05)$ and $T = 2$ years. Figure $4(a)$ displays the Saddlepoint and FFT option prices for $S_T = 1, K = \exp(0.05)$, and for maturities varying from 1 to 2 years, in steps of 0.05. One can see that for a maturity between 1.6 and 1.75 years, and particularly for $T = 1.7$, the option prices produced using all three Saddlepoint base distributions are flawed. To understand why this problem occurs for all three base distributions, we need to investigate the lower-order Saddlepoint formula given in equation (3.1) as it is the link between the three base distributions. One can see that the final Saddlepoint formula is constructed of three parts: the cdf and pdf functions of the base distribution, and a factor multiplying the pdf. The only plausible reason why the Saddlepoint formula could give a negative value, or a value greater than 1 is because the absolute size of the factor which is multiplied by the pdf function is very large, for either or both of the probabilities under the different measures (($\mathbb{P}^{\mathbb{Q}}(S_T > K)$ or ($\mathbb{P}^{\mathbb{R}}(S_T > K)$) used to form the final option price. Referring to the Saddlepoint formula in equation (3.1), one can see that the composition of this factor involves calculating "s". When this value is very close to zero, the multiplying factor becomes very large. Figure 4(b) shows that this is exactly the case when calculating $\mathbb{P}^{\mathbb{Q}}(S_T > K)$, as s, calculated using a Gaussian base distribution, hits a minimum of 0.002271 for a maturity of 1.7 years.

The same tests have been carried out for when we change the interest rate, and include a dividend yield in the calculation. Changing these parameters so that $r = 0.1, q = 0.02$, figure 5(a) shows that compared to the FFT option price, the Saddlepoint option prices produce unreliable results at both $T = 0.5$ years and $T = 0.85$ years. Figures 5(b) and 5(c), display the values of s from equation (3.1) when calculating $\mathbb{P}^{\mathbb{Q}}(S_T > K)$ and $\mathbb{P}^{\mathbb{R}}(S_T > K)$, where the values of s were calculated using a Gaussian base distribution. The two graphs confirm that at $T = 0.5$ years and $T = 0.85$ years, s hits a minimum of 0.001996 and 0.004638 respectively. (The bigger error for a maturity of 0.5 years is reflected in the smaller value of s).

This problem only seems to occur for the VG model. This can be demonstrated by applying a different set of parameters: By changing the parameters to $\theta = -1, \nu = 1.2, \sigma^2_{\text{vg}} = 0.05$ (and the corresponding C, G, M parameterisation: $C = 0.833333, G = 0.816660$, $M = 40.816660$), and using $S_{t_0} = 1, K = \exp(0.05), r = 0.05, q = 0$, figure 6($a$) shows that the Saddlepoint method provides unreliable results at $T = 0.2$ years. Figure 6(b) confirms that at this maturity, s, calculated using a Gaussian base distribution, hits a minimum of 0.006892.

Digressing slightly, it will be useful to confirm that we did not experience these problems with the CGMY parameters in our previous tests, where $Y_{\text{up}}, Y_{\text{down}} \neq 0$. Taking parameter set 6 as an example, figure 7(a) displays the vanilla call option values obtained using the FFT method and the Saddlepoint approximation method for all three base distributions for $S_T = 1, K = \exp(0.05), r = 0.05, q = 0$ and for varying maturities ranging from 0 to 2.5 years. Figure 7(b) displays the corresponding values of s, calculated using a Gaussian base distribution for varying maturities. As one can see, the Saddlepoint method using the Merton and Gaussian base distributions produce accurate results for this parameter set without any problems. (The Kou model experiences problems for larger maturities with this parameter set (i.e. higher C parameter set), which we have recognised in Section 5).

Since the option prices from Rogers and Zane (1999) do not experience this problem, it will be interesting to see how our higher-order Saddlepoint approximation using a Gaussian base distribution behaves for these VG parameters, as the higher-order formula in equation (3.3) is not a direct extension of the lower-order formula in equation (3.1), as is the case for the Merton and Kou model. We computed option prices for the original parameters in Rogers and Zane (1999), using the higher-order Saddlepoint approximation for a Gaussian base distribution, and the FFT method - to be used as a comparison for the actual values. Figure 8 displays the results. We can see that the results from the higher-order approximation are very good, as it didn't encounter the same errors in the value of s, as the lower-order approximation has - (see figure 4). Therefore, in order to overcome the obstacle of producing flawed option prices due to a very small value of s, it may be necessary to calculate the minimum value of s for varying maturities, and then switch to applying the higher-order Saddlepoint approximation for a Gaussian base distribution if necessary.

6 Extension of Carr-Madan Base

We will now consider an extension to the base used in Carr & Madan's paper "Saddlepoint Methods for Option Pricing". The base distribution used in this paper is: $Z+\frac{1}{\lambda}-Y$, where Z is a standard Gaussian random variable and Y is a positive exponential with parameter λ. We will extend this by letting Z denote a random variable following a Merton distribution.

We omit full details for the sake of brevity, since they can be found in Carr and Madan (2008). However, we do need to specify the cdf and pdf of the proposed "Merton Minus Exponential" base distribution which we do in the following Proposition and Corollary.

6.1 Merton Minus Exponential Distribution

Proposition 6.1.1. *The complementary cdf for the base $Z+\frac{1}{\lambda}-Y$ is given by:*

$$\begin{aligned}\mathbb{P}\left(Z+\frac{1}{\lambda}-Y>a\right) &= \sum_{k_1=0}^{\infty}\sum_{k_2=0}^{\infty} P_p(\lambda_1 t,k_1)P_p(\lambda_2 t,k_2)\cdot \\ &\quad \left[\left(1-\Phi\left(a-\frac{1}{\lambda},\alpha,\beta\right)\right)-e^{\frac{\beta\lambda^2}{2}-\alpha\lambda+\lambda a-1}\left(1-\Phi\left(a-\frac{1}{\lambda},\alpha-\beta\lambda,\beta\right)\right)\right],\end{aligned}$$

where $\alpha=\gamma t+k_1\mu_1+k_2\mu_2$ and $\beta=\sqrt{\sigma_{\text{Mert}}^2 t+k_1\delta_1^2+k_2\delta_2^2}$.

Proof.

$$\begin{aligned}\mathbb{P}\left(Z+\frac{1}{\lambda}-Y>a\right) &= \int_{a-\frac{1}{\lambda}}^{\infty} f_Z(z)\left(1-e^{(-\lambda(z-(a-\frac{1}{\lambda})))}\right)dz \\ &= \int_{a-\frac{1}{\lambda}}^{\infty}\left(f_Z(z)-f_Z(z)e^{(-\lambda(z-(a-\frac{1}{\lambda})))}\right)dz \\ &= \int_{a-\frac{1}{\lambda}}^{\infty} f_Z(z)dz-\int_{a-\frac{1}{\lambda}}^{\infty} f_Z(z)e^{(-\lambda(z-(a-\frac{1}{\lambda})))}dz.\end{aligned}$$

$$\begin{aligned}\int_{a-\frac{1}{\lambda}}^{\infty} f_Z(z)dz &= \int_{a-\frac{1}{\lambda}}^{\infty}\sum_{k_1=0}^{\infty}\sum_{k_2=0}^{\infty}\frac{e^{(-\lambda_1 t)}(\lambda_1 t)^{k_1}}{k_1!}\frac{e^{(-\lambda_2 t)}(\lambda_2 t)^{k_2}}{k_2!}\frac{e^{\left\{-\frac{(z-\gamma t-k_1\mu_1-k_2\mu_2)^2}{2(\sigma_{\text{Mert}}^2 t+k_1\delta_1^2+k_2\delta_2^2)}\right\}}}{\sqrt{2\pi(\sigma_{\text{Mert}}^2 t+k_1\delta_1^2+k_2\delta_2^2)}}dz \\ &= \sum_{k_1=0}^{\infty}\sum_{k_2=0}^{\infty} P_p(\lambda_1 t,k_1)P_p(\lambda_2 t,k_2)\cdot \\ &\quad \int_{a-\frac{1}{\lambda}}^{\infty}\phi\left(z,\gamma t+k_1\mu_1+k_2\mu_2,\sqrt{\sigma_{\text{Mert}}^2 t+k_1\delta_1^2+k_2\delta_2^2}\right)dz \\ &= \sum_{k_1=0}^{\infty}\sum_{k_2=0}^{\infty} P_p(\lambda_1 t,k_1)P_p(\lambda_2 t,k_2)\left(1-\Phi\left(a-\frac{1}{\lambda},\alpha,\beta\right)\right),\end{aligned}$$

where $\alpha=\gamma t+k_1\mu_1+k_2\mu_2$ and $\beta=\sqrt{\sigma_{\text{Mert}}^2 t+k_1\delta_1^2+k_2\delta_2^2}$.

$$\begin{aligned}\int_{a-\frac{1}{\lambda}}^{\infty} f_Z(z)e^{(-\lambda(z-(a-\frac{1}{\lambda})))}dz &= e^{(\lambda a-1)}\int_{a-\frac{1}{\lambda}}^{\infty} f_Z(z)e^{(-\lambda z)}dz \\ &= e^{(\lambda a-1)}\sum_{k_1=0}^{\infty}\sum_{k_2=0}^{\infty} P_p(\lambda_1 t,k_1)P_p(\lambda_2 t,k_2)\int_{a-\frac{1}{\lambda}}^{\infty} e^{(-\lambda z)}\phi\left(z,\alpha,\beta\right)dz \\ &= e^{(\lambda a-1)}\sum_{k_1=0}^{\infty}\sum_{k_2=0}^{\infty} P_p(\lambda_1 t,k_1)P_p(\lambda_2 t,k_2)\int_{a-\frac{1}{\lambda}}^{\infty} \frac{e^{(-\lambda z)}e^{-\frac{(z-\alpha)^2}{2\beta}}}{\sqrt{2\pi\beta}}dz.\end{aligned}$$

Completing the square, we obtain:

$$\begin{aligned}e^{(-\lambda z)}e^{-\frac{(z-\alpha)^2}{2\beta}} &= e^{-\frac{(z-(\alpha-\beta\lambda))^2}{2\beta}}e^{\frac{\beta^2\lambda^2-2\alpha\beta\lambda}{2\beta}} \\ &= e^{-\frac{(z-(\alpha-\beta\lambda))^2}{2\beta}}e^{\frac{\beta\lambda^2}{2}-\alpha\lambda}.\end{aligned}$$

$$\begin{aligned}\int_{a-\frac{1}{\lambda}}^{\infty} f_Z(z)e^{(-\lambda(z-(a-\frac{1}{\lambda})))}dz &= e^{\frac{\beta\lambda^2}{2}-\alpha\lambda}e^{(\lambda a-1)}\sum_{k_1=0}^{\infty}\sum_{k_2=0}^{\infty} P_p(\lambda_1 t,k_1)P_p(\lambda_2 t,k_2)\int_{a-\frac{1}{\lambda}}^{\infty} \frac{e^{-\frac{(z-(\alpha-\beta\lambda))^2}{2\beta}}}{\sqrt{2\pi\beta}}dz \\ &= e^{\frac{\beta\lambda^2}{2}-\alpha\lambda+\lambda a-1}\cdot \\ &\quad \sum_{k_1=0}^{\infty}\sum_{k_2=0}^{\infty} P_p(\lambda_1 t,k_1)P_p(\lambda_2 t,k_2)\left(1-\Phi\left(a-\frac{1}{\lambda},\alpha-\beta\lambda,\beta\right)\right).\end{aligned}$$

Simplifying this equation, the complementary cdf of this base is:

$$\begin{aligned}\mathbb{P}\left(Z+\frac{1}{\lambda}-Y>a\right) &= \sum_{k_1=0}^{\infty}\sum_{k_2=0}^{\infty} P_p(\lambda_1 t,k_1)P_p(\lambda_2 t,k_2)\cdot \\ &\left[\left(1-\Phi\left(a-\frac{1}{\lambda},\alpha,\beta\right)\right)-e^{\frac{\beta\lambda^2}{2}-\alpha\lambda+\lambda a-1}\left(1-\Phi\left(a-\frac{1}{\lambda},\alpha-\beta\lambda,\beta\right)\right)\right].\end{aligned}$$

□

Corollary 6.1.2. *The pdf of the base* $Z+\frac{1}{\lambda}-Y$ *is given by:*

$$\begin{aligned}f_{Z+\frac{1}{\lambda}-Y}(a) &= \sum_{k_1=0}^{\infty}\sum_{k_2=0}^{\infty} P_p(\lambda_1 t,k_1)P_p(\lambda_2 t,k_2)\cdot \\ &\left[\phi\left(a-\frac{1}{\lambda},\alpha,\beta\right)+\lambda e^{\frac{\beta\lambda^2}{2}-\alpha\lambda+\lambda a-1}\left(1-\Phi\left(a-\frac{1}{\lambda},\alpha-\beta\lambda,\beta\right)\right)\right. \\ &\left.\qquad -e^{\frac{\beta\lambda^2}{2}-\alpha\lambda+\lambda a-1}\phi\left(a-\frac{1}{\lambda},\alpha-\beta\lambda,\beta\right)\right].\end{aligned}$$

Proof.

$$
\begin{aligned}
f(a) &= \frac{d\left(\mathbb{P}\left(Z+\frac{1}{\lambda}-Y<a\right)\right)}{da} \\
&= -\frac{d\left(\mathbb{P}\left(Z+\frac{1}{\lambda}-Y>a\right)\right)}{da} \\
&= -\sum_{k_1=0}^{\infty}\sum_{k_2=0}^{\infty} P_p(\lambda_1 t, k_1) P_p(\lambda_2 t, k_2)\cdot \\
& \quad \frac{d\left[\left(1-\Phi\left(a-\frac{1}{\lambda},\alpha,\beta\right)\right)-e^{\frac{\beta\lambda^2}{2}-\alpha\lambda+\lambda a-1}\left(1-\Phi\left(a-\frac{1}{\lambda},\alpha-\beta\lambda,\beta\right)\right)\right]}{da} \\
&= \sum_{k_1=0}^{\infty}\sum_{k_2=0}^{\infty} P_p(\lambda_1 t, k_1) P_p(\lambda_2 t, k_2).\left[\phi\left(a-\frac{1}{\lambda},\alpha,\beta\right)\right. \\
&+ \lambda e^{\frac{\beta\lambda^2}{2}-\alpha\lambda+\lambda a-1}\left(1-\Phi\left(a-\frac{1}{\lambda},\alpha-\beta\lambda,\beta\right)\right) \\
&- \left. e^{\frac{\beta\lambda^2}{2}-\alpha\lambda+\lambda a-1}\phi\left(a-\frac{1}{\lambda},\alpha-\beta\lambda,\beta\right)\right].
\end{aligned}
$$

□

6.2 Results

We have computed some results using the Gaussian Minus Exponential (henceforth GME) base distribution, and the Merton Minus Exponential (henceforth MME) base distribution.

6.2.1 Comparison Against Black Scholes

The first set of results are computed for where the log of stock price follows the Black Scholes model. We compare results obtained from using our GME base to the values in Carr and Madan (2008), and the correct option prices obtained from the Black Scholes pricing formula.
The option parameters used are: maturity $T = 0.5, 1$; initial stock price $S_{t_0} = 100$; risk-free rate $r = 0.03$; dividend yield $q = 0$; varying strikes, K; and GME volatility $\sigma = 0.25$.
The results can be found in table 13 in Appendix B.

Carr and Madan (2008) prove that the GME base distribution is a shift and scale of the distribution that we are approximating, and therefore, by proposition 3.1.3, we would expect the results from our Saddlepoint approximation using the GME base distribution to be exact. Our results show that the GME base distribution performs as expected, as the results match perfectly with those obtained from the Black Scholes pricing formula. The results from Carr and Madan (2008) are slightly different - but this may be due to the values of the strikes that are used in that paper, as they have been rounded.

6.2.2 CGMY Results

The second set of results are computed for where the log of the stock price follows a CGMY process. We compare results obtained from using our GME and MME base distributions

with the correct values from the numerical integration and FFT methods.
The CGMY parameters used are parameter sets 1, 7 and 10.
The option parameters used are: maturity $T = 1$; initial stock price $S_{t_0} = 100$; risk-free rate $r = 0.03$; dividend yield $q = 0$; strike K takes values from 60 to 140, in steps of 10; and CGMY volatility $\sigma_{\text{CGMY}} = 0.2$.
The results can be found in table 14 in Appendix B.

We can see that the GME base distribution has performed very well as all of the results are within 0.14 of the integral values. The MME base hasn't made the improvement upon the results of the GME base as we had hoped, but the majority of the results are within 0.33 of the integration values. The largest differences being for ITM options for parameter set 1. We note that excluding the ITM options from parameter set 1 for the MME base distribution, the results from both the GME and MME base distributions are all less than the true option price. Therefore, in the spirit of suggesting avenues for future research, a correction term might be added onto the option prices from both bases to produce a more accurate result.

7 Conclusions

Lévy process models were introduced to compensate for the drawbacks of the standard Black-Scholes model. However, since the probability density function is rarely known in closed form, pricing vanilla options under Lévy processes require numerical methods.

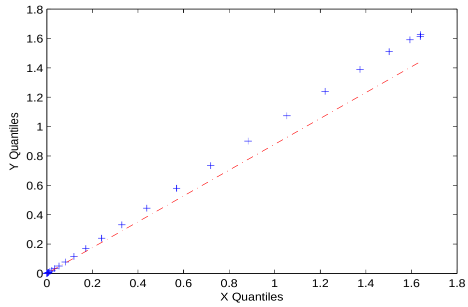
The Saddlepoint approximation method uses a model's cumulant generating function, which is always available, to compute probabilities, and hence to compute option prices. It produces results that are far superior to the numerical integration method with respect to speed, and produces reliable and accurate results where the FFT option pricing technique breaks down.

In this dissertation, we've shown that the Merton (1976) and Kou (2002) models both generate good base distributions to use in the Saddlepoint approximation method to price options under the CGMY model. These base distributions outperform the commonly used Gaussian base distribution in terms of accuracy, depending on the CGMY parameters. We've also demonstrated that results obtained from using the higher-order approximation reveal a distinct pattern in the strikes and CGMY parameters for which it outperforms results from using the lower-order approximation.

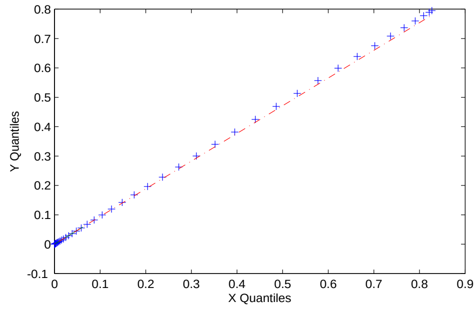
Further research that could be done is to incorporate the higher-order Saddlepoint formula for the Gaussian Minus Exponential and Merton Minus Exponential distributions into Carr and Madan's (2008) method of using a single probability to value vanilla options.

Appendix A: QQ Plots

- Results for $S_{t_0} = 1, K = 1, r = 0.05, q = 0.02, \sigma_{\text{CGMY}} = 0.2, T = 1$.

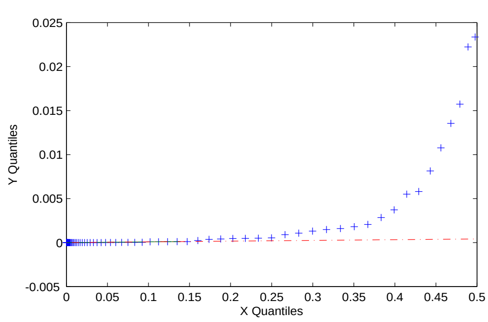


(a) Set 1 QQ plot for Merton-Kou

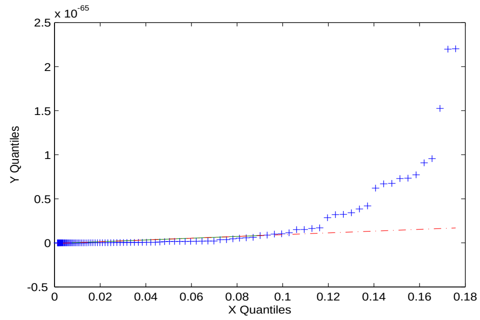


(b) Set 6 QQ plot for Merton-Kou

Figure 1: QQ Plots for Merton & Kou distributions where $0 < Y_{\text{up}}, Y_{\text{down}} < 1$.

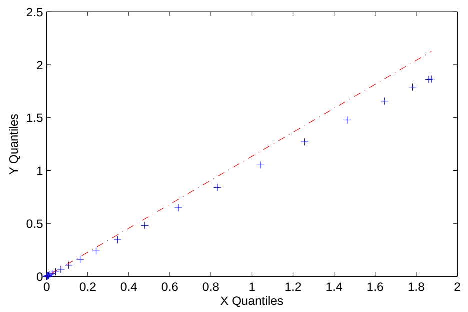


(a) Set 10 QQ plot for Merton-Kou

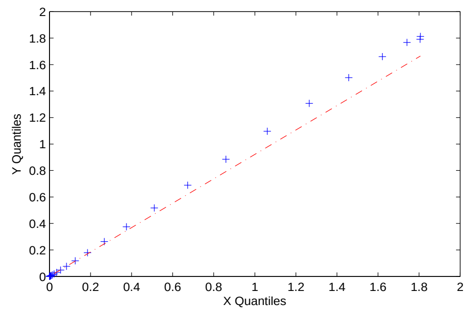


(b) Set 11 QQ plot for Merton-Kou

Figure 2: QQ Plots for Merton & Kou distributions where $1 < Y_{\text{up}}, Y_{\text{down}} < 2$.



(a) Set 12 QQ plot for Merton-Kou



(b) Set 15 QQ plot for Merton-Kou

Figure 3: QQ Plots for Merton & Kou distributions where $Y_{\text{up}}, Y_{\text{down}} < 0$.

Appendix B : Results Tables

Table 1: BCON Style Option Values: $S_{t_0} = 100, r = 0.05, q = 0.02, \sigma_{\mathrm{CGMY}} = 0, T = 1$

Set	K	Merton-0	Merton-$\hat{t}$	Kou-0	Kou-$\hat{t}$	Gaussian	Integral
1	60	0.924844	0.929460	0.933284	0.927562	0.932613	0.932116
1	70	0.910637	0.894389	0.912547	0.901637	0.910250	0.909092
1	80	0.902655	0.828168	0.874751	0.855301	0.867519	0.865254
1	90	0.844337	0.722139	0.807547	0.772755	0.787612	0.784256
1	100	0.585566	0.566565	0.630545	0.619620	0.634383	0.633962
1	110	0.305302	0.328448	0.311216	0.330619	0.336783	0.336612
1	120	0.085503	0.114061	0.084151	0.095703	0.092173	0.084157
1	130	0.024790	0.034178	0.024078	0.029664	0.026639	0.025401
1	140	0.008904	0.010118	0.008351	0.010598	0.008986	0.008749
2	60	0.876225	0.873040	0.877253	0.876536	0.876419	0.875543
2	70	0.815799	0.810353	0.813676	0.812507	0.812069	0.810895
2	80	0.732423	0.723616	0.725202	0.723641	0.722957	0.721651
2	90	0.621678	0.615735	0.615267	0.614134	0.613321	0.612123
2	100	0.493780	0.495448	0.492926	0.493313	0.492470	0.491590
2	110	0.368430	0.375327	0.371450	0.373454	0.372651	0.372162
2	120	0.260160	0.267638	0.263560	0.266248	0.265544	0.265355
2	130	0.174731	0.180448	0.176990	0.179434	0.178878	0.178824
2	140	0.112269	0.116054	0.113448	0.115270	0.114877	0.114841
3	60	0.887311	0.882543	0.889205	0.888214	0.888267	0.887438
3	70	0.833741	0.823969	0.831184	0.829509	0.829192	0.828010
3	80	0.755683	0.739864	0.745818	0.743337	0.742741	0.741353
3	90	0.641096	0.630440	0.633441	0.631428	0.630764	0.629423
3	100	0.501356	0.503148	0.502187	0.502631	0.502102	0.501019
3	110	0.363987	0.372713	0.368958	0.371772	0.371487	0.370673
3	120	0.247656	0.255911	0.251651	0.254969	0.254929	0.254233
3	130	0.158705	0.164182	0.160726	0.163281	0.163395	0.162744
3	140	0.096712	0.099974	0.097510	0.099139	0.099295	0.098756
4	60	0.919292	0.913843	0.920495	0.912091	0.919590	0.916015
4	70	0.874886	0.851911	0.872083	0.858386	0.869853	0.864610
4	80	0.787813	0.758811	0.780302	0.767804	0.778518	0.772488
4	90	0.645988	0.631698	0.642395	0.636712	0.644330	0.638944
4	100	0.480767	0.481309	0.482645	0.482866	0.487083	0.483470
4	110	0.330675	0.336641	0.333908	0.336497	0.337926	0.336309
4	120	0.213825	0.220781	0.216369	0.219618	0.219193	0.219070
4	130	0.132660	0.139430	0.134157	0.137490	0.136059	0.136744
4	140	0.080586	0.086791	0.081257	0.084370	0.082518	0.083490
5	60	0.858734	0.861119	0.860142	0.860539	0.858942	0.859103
5	70	0.796510	0.794720	0.793012	0.793057	0.790761	0.791166
5	80	0.714029	0.706627	0.704820	0.704304	0.701419	0.702219
5	90	0.606156	0.600936	0.599300	0.598693	0.595527	0.596782
5	100	0.483490	0.485954	0.483852	0.484290	0.481367	0.482868
5	110	0.364824	0.372651	0.369553	0.371493	0.369423	0.370621
5	120	0.263132	0.271593	0.267927	0.270562	0.269700	0.270024
5	130	0.183047	0.189615	0.186192	0.188518	0.188715	0.188112
5	140	0.123997	0.128324	0.125636	0.127252	0.128029	0.126974
6	60	0.797682	0.798179	0.767771	0.774361	0.796185	0.795947
6	70	0.716442	0.716150	0.689358	0.692752	0.713962	0.713717
6	80	0.625365	0.624672	0.601461	0.602720	0.622577	0.622350
6	90	0.530949	0.530693	0.510782	0.510945	0.528875	0.528676
6	100	0.439619	0.440266	0.423288	0.423148	0.438804	0.438630
6	110	0.356295	0.357757	0.343360	0.343387	0.356655	0.356494
6	120	0.283762	0.285663	0.273539	0.273925	0.284881	0.284725
6	130	0.222886	0.224853	0.214724	0.215478	0.224331	0.224175
6	140	0.173211	0.174997	0.166604	0.167649	0.174676	0.174521
7	60	0.590332	0.590259	0.018217	0.018191	0.590212	0.590039
7	70	0.518334	0.518309	0.015876	0.015850	0.518263	0.518110
7	80	0.454129	0.454150	0.013732	0.013765	0.454106	0.453975
7	90	0.397640	0.397701	0.011820	0.011943	0.397659	0.397550
7	100	0.348329	0.348423	0.010146	0.010368	0.348383	0.348295
7	110	0.305474	0.305593	0.008697	0.009013	0.305554	0.305485
7	120	0.268311	0.268447	0.007452	0.007850	0.268410	0.268357
7	130	0.236104	0.236252	0.006388	0.006853	0.236217	0.236177
7	140	0.208186	0.208340	0.005480	0.005997	0.208306	0.208279
8	60	0.845427	0.851549	0.849413	0.850484	0.848399	0.848147
8	70	0.789889	0.791624	0.786880	0.787920	0.785097	0.784746
8	80	0.723403	0.715702	0.708326	0.708998	0.705545	0.705132
8	90	0.633296	0.625645	0.616228	0.616359	0.612461	0.612075
8	100	0.520234	0.525156	0.514314	0.514432	0.510289	0.510082
8	110	0.401149	0.419669	0.407555	0.409049	0.404797	0.404994
8	120	0.292934	0.315916	0.302965	0.306833	0.302469	0.303373
8	130	0.201761	0.221084	0.208683	0.214383	0.209727	0.211686
8	140	0.127737	0.141583	0.131034	0.137303	0.132153	0.135357
9	60	0.898072	0.895919	0.904379	0.905137	0.903352	0.903292
9	70	0.861531	0.847118	0.859662	0.860367	0.857075	0.857063
9	80	0.813853	0.774129	0.790202	0.789458	0.784367	0.784714
9	90	0.709091	0.671898	0.689730	0.686349	0.679453	0.681006
9	100	0.536315	0.537868	0.548412	0.548768	0.540397	0.544748
9	110	0.356193	0.380382	0.374601	0.384527	0.376188	0.383589
9	120	0.206850	0.230527	0.214123	0.224717	0.219874	0.222474
9	130	0.105563	0.124427	0.107151	0.114000	0.112745	0.109231
9	140	0.052483	0.064952	0.052387	0.056204	0.056072	0.053688

Continued on next page

Table 1 – continued from previous page

set	K	Merton-0	Merton-$\hat{t}$	Kou-0	Kou-$\hat{t}$	Gaussian	integral
10	60	0.615868	0.615824	0.000477	0.001369	0.615932	0.615696
10	70	0.540194	0.540177	-0.000092	0.000333	0.540271	0.540056
10	80	0.471966	0.471968	-0.000154	0.000547	0.472051	0.471857
10	90	0.411552	0.411569	-0.000076	0.000139	0.411640	0.411469
10	100	0.358646	0.358674	0.000121	0.000173	0.358734	0.358584
10	110	0.312624	0.312658	0.000038	0.000114	0.312710	0.312579
10	120	0.272745	0.272783	0.000041	0.000062	0.272828	0.272713
10	130	0.238257	0.238298	0.000006	0.000035	0.238336	0.238237
10	140	0.208455	0.208496	0.000030	0.000009	0.208529	0.208442
11	60	0.299483	0.299484	0.000000	0.000000	0.299487	0.299479
11	70	0.268435	0.268436	0.000000	0.000000	0.268438	0.268431
11	80	0.242843	0.242844	0.000000	0.000000	0.242847	0.242840
11	90	0.221366	0.221368	0.000000	0.000000	0.221370	0.221364
11	100	0.203077	0.203078	0.000000	0.000000	0.203080	0.203075
11	110	0.187311	0.187313	0.000000	0.000000	0.187315	0.187309
11	120	0.173581	0.173582	0.000000	0.000000	0.173584	0.173579
11	130	0.161518	0.161519	0.000000	0.000000	0.161521	0.161516
11	140	0.150838	0.150839	0.000000	0.000000	0.150841	0.150836
12	60	0.939466	0.943008	0.944742	0.944879	0.943557	0.781965
12	70	0.935543	0.928524	0.936697	0.936450	0.933522	0.992924
12	80	0.934165	0.891447	0.921686	0.918967	0.913823	1.073402
12	90	0.938264	0.807007	0.895178	0.880922	0.876156	0.956567
12	100	0.721817	0.607683	0.826565	0.764095	0.788731	0.821607
12	110	0.052904	0.162514	0.050067	0.094094	0.089617	0.013092
12	120	0.009115	0.033475	0.017817	0.025121	0.025299	-0.071724
12	130	0.009331	0.010276	0.007549	0.009679	0.009929	-0.186301
12	140	0.006474	0.004365	0.003452	0.004277	0.004360	-0.043865
13	60	0.904027	0.908222	0.913868	0.915427	0.912513	0.903710
13	70	0.882788	0.868804	0.879309	0.882041	0.876040	0.895339
13	80	0.864896	0.806021	0.825284	0.829132	0.817630	0.774586
13	90	0.787011	0.710153	0.746327	0.749178	0.727838	0.733857
13	100	0.570690	0.566497	0.626360	0.625722	0.588145	0.617320
13	110	0.326110	0.360003	0.356161	0.375014	0.363629	0.419264
13	120	0.134315	0.171349	0.126386	0.126614	0.155145	0.129851
13	130	0.055814	0.079310	0.056862	0.054680	0.067458	0.051882
13	140	0.029350	0.039257	0.028806	0.027068	0.032487	0.031403
14	60	0.921402	0.918755			0.924214	0.849831
14	70	0.915989	0.887621			0.899337	0.613249
14	80	0.914095	0.834784			0.859267	0.688169
14	90	0.884369	0.745140			0.795619	0.823548
14	100	0.609025	0.582828			0.668576	0.863742
14	110	0.205320	0.279699			0.229279	-0.027328
14	120	0.037143	0.119771			0.083749	0.062299
14	130	0.019833	0.060880			0.042905	-0.080939
14	140	0.017414	0.033781			0.024269	-0.128533
15	60	0.926897	0.921721			0.926910	0.557685
15	70	0.923802	0.893662			0.905304	0.799862
15	80	0.922924	0.845897			0.871489	0.694428
15	90	0.915092	0.764017			0.821247	0.748462
15	100	0.666037	0.611958			0.744851	0.792465
15	110	0.136213	0.228399			0.149280	0.053365
15	120	0.012769	0.072051			0.043871	0.000330
15	130	0.008651	0.030138			0.019753	-0.126446
15	140	0.007585	0.014012			0.009782	0.106096

Table 2: BCON Style Option Values: $S_{t_0} = 100, r = 0.05, q = 0.02, \sigma_{\mathrm{CGMY}} = 0.2, T = 1$

Set	K	Merton-0	Merton-$\hat{t}$	Merton-Higher	Kou-0	Kou-$\hat{t}$	Kou-Higher	Gaussian	Integral
1	60	0.919836	0.904884	0.918485	0.917989	0.907512	0.908276	0.915794	0.913518
1	70	0.881673	0.846056	0.873650	0.872068	0.856455	0.863649	0.871039	0.863706
1	80	0.793306	0.755106	0.769431	0.781871	0.768527	0.784749	0.788469	0.774257
1	90	0.650289	0.633708	0.621452	0.647091	0.642105	0.656781	0.661087	0.644932
1	100	0.490457	0.490901	0.479758	0.493309	0.493435	0.496839	0.506010	0.494212
1	110	0.345259	0.348607	0.346741	0.347749	0.348744	0.346050	0.355086	0.348721
1	120	0.226740	0.228952	0.229431	0.227858	0.228613	0.225224	0.231092	0.228416
1	130	0.139549	0.141014	0.141452	0.140083	0.140668	0.138136	0.141157	0.140479
1	140	0.081318	0.082603	0.082890	0.081706	0.082275	0.080654	0.081840	0.082116
2	60	0.846786	0.842698	0.844077	0.845584	0.843949	0.843398	0.845152	0.843707
2	70	0.770429	0.765966	0.766512	0.767863	0.766178	0.767550	0.767756	0.765793
2	80	0.674044	0.670641	0.669605	0.671270	0.670088	0.672526	0.671885	0.669622
2	90	0.566426	0.565169	0.562615	0.564717	0.564302	0.566345	0.566088	0.563818
2	100	0.458526	0.459253	0.456186	0.458150	0.458388	0.459054	0.459960	0.457939
2	110	0.359242	0.361051	0.358443	0.359728	0.360331	0.359503	0.361569	0.359949
2	120	0.273705	0.275754	0.273969	0.274496	0.275209	0.273354	0.276084	0.274904
2	130	0.203670	0.205496	0.204434	0.204432	0.205105	0.202835	0.205653	0.204872
2	140	0.148601	0.150058	0.149478	0.149210	0.149786	0.147583	0.150076	0.149613
3	60	0.859504	0.853243	0.856595	0.858138	0.855913	0.855372	0.857563	0.855946
3	70	0.786092	0.779006	0.781480	0.782882	0.780502	0.782289	0.782766	0.780386
3	80	0.688690	0.683356	0.683165	0.685330	0.683636	0.686750	0.686276	0.683388
3	90	0.576502	0.574519	0.571751	0.574705	0.574091	0.576635	0.576720	0.573759
3	100	0.462612	0.463429	0.459896	0.462524	0.462780	0.463543	0.465052	0.462426
3	110	0.357812	0.359930	0.357130	0.358638	0.359324	0.358313	0.361056	0.358998
3	120	0.268197	0.270455	0.268753	0.269207	0.269983	0.267909	0.271156	0.269712
3	130	0.195760	0.197648	0.196770	0.196611	0.197312	0.194948	0.198009	0.197101
3	140	0.139777	0.141206	0.140793	0.140397	0.140975	0.138841	0.141321	0.140819

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Table 2 – continued from previous page

Set	K	Merton-0	Merton-$\hat{t}$	Merton-Higher	Kou-0	Kou-$\hat{t}$	Kou-Higher	Gaussian	Integral
4	60	0.887147	0.875719	0.877903	0.885750	0.878653	0.883722	0.885779	0.881222
4	70	0.810934	0.799707	0.800949	0.808440	0.802883	0.809667	0.809715	0.804633
4	80	0.702629	0.696369	0.694847	0.700638	0.697819	0.703459	0.703059	0.698611
4	90	0.577599	0.575698	0.573392	0.576884	0.576063	0.578979	0.579515	0.576296
4	100	0.453332	0.453744	0.452140	0.453556	0.453733	0.454110	0.455656	0.453726
4	110	0.342271	0.343590	0.342873	0.342905	0.343496	0.342307	0.344265	0.343406
4	120	0.250461	0.252068	0.251909	0.251186	0.251962	0.250160	0.251931	0.251846
4	130	0.178889	0.180586	0.180658	0.179567	0.180456	0.178685	0.179914	0.180324
4	140	0.125507	0.127237	0.127332	0.126084	0.127053	0.125645	0.126216	0.126901
5	60	0.830968	0.829230	0.829103	0.829462	0.828809	0.828033	0.828677	0.828218
5	70	0.753923	0.750679	0.750889	0.750723	0.749832	0.750557	0.750065	0.749244
5	80	0.659420	0.656416	0.655668	0.656100	0.655372	0.656992	0.656027	0.654853
5	90	0.555499	0.554402	0.552114	0.553630	0.553371	0.554758	0.554327	0.552939
5	100	0.452190	0.453171	0.450159	0.452037	0.452275	0.452615	0.453310	0.451925
5	110	0.357514	0.359740	0.357075	0.358449	0.359020	0.358185	0.359930	0.358739
5	120	0.275939	0.278519	0.276672	0.277253	0.277967	0.276332	0.278637	0.277743
5	130	0.208868	0.211270	0.210169	0.210131	0.210855	0.208940	0.211265	0.210678
5	140	0.155694	0.157714	0.157103	0.156737	0.157405	0.155622	0.157594	0.157266
6	60	0.769033	0.768622	0.767558	0.739925	0.745205	0.739621	0.767904	0.767363
6	70	0.684239	0.683645	0.682565	0.658496	0.661122	0.658776	0.683012	0.682372
6	80	0.594438	0.593995	0.592856	0.572192	0.573140	0.572749	0.593500	0.592822
6	90	0.505845	0.505790	0.504597	0.487036	0.487102	0.487539	0.505433	0.504775
6	100	0.423090	0.423465	0.422294	0.407435	0.407174	0.407673	0.423219	0.422622
6	110	0.348965	0.349669	0.348615	0.336045	0.335791	0.335953	0.349503	0.348991
6	120	0.284667	0.285556	0.284676	0.274032	0.273953	0.273653	0.285441	0.285023
6	130	0.230244	0.231189	0.230499	0.221481	0.221635	0.220911	0.231104	0.230778
6	140	0.185033	0.185944	0.185430	0.177793	0.178173	0.177136	0.185878	0.185633
7	60	0.577620	0.577564	0.577361	0.017738	0.017741	0.017742	0.577555	0.577381
7	70	0.507321	0.507305	0.507119	0.015462	0.015450	0.015465	0.507294	0.507141
7	80	0.445057	0.445079	0.444912	0.013399	0.013430	0.013401	0.445066	0.444934
7	90	0.390507	0.390561	0.390414	0.011573	0.011675	0.011574	0.390546	0.390435
7	100	0.343001	0.343080	0.342953	0.009979	0.010162	0.009979	0.343063	0.342973
7	110	0.301752	0.301850	0.301741	0.008601	0.008863	0.008599	0.301831	0.301759
7	120	0.265974	0.266087	0.265993	0.007416	0.007748	0.007413	0.266066	0.266010
7	130	0.234938	0.235059	0.234980	0.006400	0.006791	0.006397	0.235037	0.234994
7	140	0.207988	0.208114	0.208048	0.005531	0.005967	0.005527	0.208092	0.208060
8	60	0.824345	0.823426	0.822328	0.822972	0.822674	0.821026	0.821854	0.821324
8	70	0.755828	0.751908	0.752272	0.750691	0.750014	0.750354	0.749371	0.748497
8	80	0.671265	0.666525	0.666002	0.664552	0.663809	0.665856	0.663560	0.662289
8	90	0.574336	0.572338	0.568216	0.569635	0.569308	0.571712	0.569551	0.567912
8	100	0.473460	0.475559	0.468232	0.472304	0.472682	0.473820	0.473366	0.471479
8	110	0.377418	0.382339	0.374540	0.378907	0.379906	0.378808	0.380864	0.378919
8	120	0.292062	0.297684	0.291684	0.294495	0.295796	0.292521	0.296822	0.295017
8	130	0.219952	0.224811	0.221147	0.222151	0.223437	0.218785	0.224354	0.222841
8	140	0.161430	0.165034	0.163127	0.162987	0.164075	0.159040	0.164778	0.163632
9	60	0.880140	0.869869	0.878259	0.878479	0.876187	0.874491	0.877078	0.876198
9	70	0.819596	0.803635	0.814831	0.813249	0.809745	0.811312	0.812028	0.809909
9	80	0.726532	0.712217	0.718046	0.719543	0.716391	0.720880	0.720285	0.716595
9	90	0.606235	0.600219	0.597004	0.603425	0.602062	0.606339	0.606830	0.602130
9	100	0.477465	0.478810	0.472315	0.478744	0.479065	0.480302	0.483428	0.478943
9	110	0.357738	0.361900	0.357721	0.360369	0.361453	0.359549	0.364450	0.361206
9	120	0.256425	0.260606	0.259146	0.258733	0.259949	0.256513	0.261328	0.259656
9	130	0.176648	0.180252	0.179980	0.178358	0.179540	0.176069	0.179574	0.179240
9	140	0.117652	0.120856	0.120727	0.118928	0.120098	0.117371	0.119266	0.119802
10	60	0.601679	0.601646	0.601507	0.002488	-0.000307	0.002503	0.601744	0.601540
10	70	0.527854	0.527843	0.527714	0.003309	0.003657	0.003330	0.527928	0.527742
10	80	0.461829	0.461833	0.461717	0.001623	0.000560	0.001631	0.461908	0.461741
10	90	0.403657	0.403673	0.403569	0.000844	0.000386	0.000846	0.403738	0.403590
10	100	0.352854	0.352878	0.352786	0.000667	-0.000018	0.000668	0.352934	0.352803
10	110	0.308709	0.308737	0.308656	0.000700	0.000802	0.000699	0.308786	0.308671
10	120	0.270447	0.270479	0.270407	0.000206	-0.000042	0.000205	0.270520	0.270420
10	130	0.237318	0.237351	0.237289	0.000185	0.000179	0.000184	0.237387	0.237299
10	140	0.208632	0.208666	0.208611	0.000200	0.000090	0.000198	0.208697	0.208620
11	60	0.296578	0.296579	0.296574	0.000000	0.000000	0.000000	0.296582	0.296574
11	70	0.265894	0.265895	0.265891	0.000000	0.000000	0.000000	0.265898	0.265891
11	80	0.240611	0.240612	0.240608	0.000000	0.000000	0.000000	0.240615	0.240608
11	90	0.219398	0.219399	0.219395	0.000000	0.000000	0.000000	0.219401	0.219395
11	100	0.201334	0.201336	0.201332	0.000000	0.000000	0.000000	0.201338	0.201332
11	110	0.185764	0.185766	0.185762	0.000000	0.000000	0.000000	0.185768	0.185762
11	120	0.172204	0.172205	0.172202	0.000000	0.000000	0.000000	0.172207	0.172202
11	130	0.160290	0.160291	0.160288	0.000000	0.000000	0.000000	0.160293	0.160288
11	140	0.149740	0.149741	0.149739	0.000000	0.000000	0.000000	0.149743	0.149739
12	60	0.938699	0.929170	0.940941	0.936427	0.931420	0.931366	0.934075	0.935095
12	70	0.916215	0.888038	0.920157	0.904367	0.894909	0.894053	0.906918	0.899957
12	80	0.839723	0.800727	0.822339	0.820665	0.811500	0.816985	0.840575	0.814806
12	90	0.679864	0.661880	0.631627	0.673034	0.670078	0.680984	0.699487	0.671396
12	100	0.493898	0.493613	0.484040	0.495816	0.495779	0.497268	0.508825	0.496060
12	110	0.328190	0.329711	0.331055	0.329442	0.329649	0.328763	0.332974	0.329630
12	120	0.198116	0.200368	0.203300	0.199312	0.199678	0.198209	0.198490	0.199577
12	130	0.109001	0.113166	0.118155	0.111242	0.112113	0.109552	0.108206	0.111956
12	140	0.054676	0.060268	0.064795	0.057813	0.059389	0.056821	0.054031	0.059201
13	60	0.895840	0.884593	0.895853	0.893114	0.892849	0.890877	0.890602	0.892166
13	70	0.849078	0.825287	0.847009	0.838160	0.836387	0.836325	0.835367	0.836268
13	80	0.762005	0.737219	0.753618	0.750472	0.748085	0.750583	0.751351	0.748571
13	90	0.632838	0.621452	0.618815	0.629915	0.628694	0.631817	0.636812	0.629242
13	100	0.487452	0.488756	0.479359	0.491571	0.491722	0.492661	0.500877	0.491976
13	110	0.352602	0.357585	0.354547	0.357253	0.357893	0.356476	0.363977	0.357885
13	120	0.240381	0.245192	0.245888	0.243612	0.244314	0.242136	0.246372	0.244172
13	130	0.155049	0.159801	0.160179	0.157593	0.158372	0.156530	0.157654	0.158173

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Table 2 – continued from previous page

Set	K	Merton-0	Merton-$\hat{t}$	Merton-Higher	Kou-0	Kou-$\hat{t}$	Kou-Higher	Gaussian	Integral
13	140	0.095763	0.100589	0.099532	0.097991	0.098859	0.097778	0.097005	0.098639
14	60	0.918675	0.898883	0.919970	0.912737	0.912497	0.914008	0.907851	0.913208
14	70	0.886648	0.847219	0.888629	0.869793	0.871309	0.871559	0.865889	0.870935
14	80	0.804229	0.762250	0.797870	0.786320	0.788754	0.787045	0.794320	0.787576
14	90	0.658911	0.639257	0.632098	0.655371	0.656534	0.653791	0.677720	0.655709
14	100	0.489370	0.489922	0.476825	0.495739	0.495707	0.495010	0.514894	0.495446
14	110	0.336245	0.341673	0.347726	0.341105	0.340777	0.342056	0.345153	0.340827
14	120	0.210379	0.220908	0.226660	0.217148	0.216418	0.218287	0.209607	0.216642
14	130	0.120956	0.136204	0.130514	0.130775	0.129524	0.130598	0.120115	0.129884
14	140	0.067021	0.082307	0.069259	0.076106	0.074703	0.074628	0.068446	0.075156
15	60	0.924961	0.903392	0.926467	0.918282	0.916841	0.921591	0.912466	0.919434
15	70	0.898319	0.855528	0.902286	0.879752	0.883617	0.885298	0.875566	0.882785
15	80	0.821767	0.774356	0.820337	0.799927	0.807720	0.803659	0.811711	0.803681
15	90	0.675604	0.651862	0.645063	0.669689	0.673610	0.665081	0.701520	0.670907
15	100	0.497942	0.498031	0.479022	0.505324	0.505310	0.502588	0.533605	0.504494
15	110	0.338844	0.341634	0.346445	0.342387	0.341892	0.344325	0.353207	0.341893
15	120	0.208789	0.212956	0.219655	0.211925	0.211082	0.214758	0.210394	0.211328
15	130	0.115872	0.123562	0.127603	0.122784	0.120815	0.124678	0.114257	0.121182
15	140	0.059224	0.068175	0.064588	0.068071	0.065213	0.065677	0.058141	0.065683

Table 3: BCON Style Option Values: $S_{t_0} = 100, r = 0.05, q = 0.02, \sigma_{\mathrm{CGMY}} = 0.2, T = 2$

Set	K	Merton-0	Merton-$\hat{t}$	Merton-Higher	Kou-0	Kou-$\hat{t}$	Kou-Higher	Gaussian	Integral
1	60	0.825396	0.812395	0.821358	0.822077	0.817935	0.818869	0.822133	0.819105
1	70	0.759079	0.745671	0.751776	0.754205	0.750297	0.753891	0.756194	0.751282
1	80	0.669402	0.660460	0.660634	0.665630	0.663184	0.667670	0.669748	0.663795
1	90	0.566640	0.563256	0.559621	0.565228	0.564336	0.567546	0.570400	0.564588
1	100	0.462873	0.463122	0.459198	0.463206	0.463271	0.464419	0.468152	0.463285
1	110	0.366844	0.368495	0.365884	0.367831	0.368269	0.367791	0.371828	0.368167
1	120	0.283297	0.285061	0.283699	0.284264	0.284751	0.283417	0.287163	0.284613
1	130	0.213979	0.215400	0.214776	0.214706	0.215122	0.213534	0.216661	0.214990
1	140	0.158639	0.159669	0.159400	0.159129	0.159453	0.157957	0.160378	0.159343
2	60	0.717018	0.716652	0.715742	0.689014	0.698725	0.688857	0.716367	0.715685
2	70	0.640452	0.640062	0.639096	0.616087	0.621719	0.616353	0.639849	0.639079
2	80	0.561751	0.561488	0.560488	0.540876	0.543413	0.541340	0.561354	0.560557
2	90	0.485282	0.485236	0.484235	0.467630	0.467966	0.468072	0.485169	0.484395
2	100	0.414032	0.414215	0.413257	0.399251	0.398131	0.399524	0.414197	0.413481
2	110	0.349716	0.350086	0.349212	0.337417	0.335417	0.337454	0.350097	0.349460
2	120	0.293044	0.293536	0.292773	0.282842	0.280388	0.282645	0.293562	0.293012
2	130	0.244023	0.244575	0.243932	0.235569	0.232957	0.235175	0.244605	0.244142
2	140	0.202221	0.202783	0.202258	0.195211	0.192637	0.194675	0.202811	0.202429
3	60	0.732977	0.732306	0.731449	0.716652	0.720349	0.716376	0.732183	0.731344
3	70	0.656100	0.655417	0.654429	0.641604	0.643604	0.641877	0.655383	0.654402
3	80	0.575182	0.574721	0.573619	0.562638	0.563491	0.563179	0.574782	0.573747
3	90	0.495356	0.495243	0.494084	0.484750	0.484899	0.485273	0.495375	0.494361
3	100	0.420363	0.420590	0.419463	0.411540	0.411308	0.411858	0.420762	0.419824
3	110	0.352472	0.352954	0.351934	0.345198	0.344802	0.345233	0.353137	0.352308
3	120	0.292723	0.293352	0.292483	0.286744	0.286317	0.286506	0.293525	0.292817
3	130	0.241264	0.241947	0.241241	0.236345	0.235964	0.235891	0.242099	0.241512
3	140	0.197680	0.198351	0.197798	0.193620	0.193321	0.193023	0.198476	0.198001
4	60	0.760216	0.758288	0.757943	0.758058	0.755666	0.757916	0.760037	0.758511
4	70	0.678111	0.676815	0.676150	0.676132	0.674699	0.676578	0.678216	0.676745
4	80	0.589257	0.588582	0.587830	0.587608	0.586909	0.588203	0.589662	0.588384
4	90	0.501121	0.500936	0.500252	0.499829	0.499643	0.500273	0.501733	0.500709
4	100	0.418929	0.419074	0.418529	0.417948	0.418093	0.418128	0.419632	0.418867
4	110	0.345610	0.345948	0.345554	0.344869	0.345214	0.344801	0.346310	0.345779
4	120	0.282287	0.282721	0.282460	0.281723	0.282179	0.281482	0.282930	0.282593
4	130	0.228882	0.229350	0.229192	0.228445	0.228954	0.228118	0.229441	0.229258
4	140	0.184620	0.185086	0.185005	0.184273	0.184799	0.183935	0.185091	0.185024
5	60	0.697289	0.697103	0.696205	0.674962	0.679725	0.674911	0.696312	0.696009
5	70	0.621807	0.621512	0.620622	0.602259	0.604724	0.602484	0.620830	0.620447
5	80	0.545810	0.545596	0.544689	0.528948	0.529886	0.529287	0.545052	0.544618
5	90	0.472919	0.472915	0.469834	0.458569	0.458581	0.461010	0.472506	0.472052
5	100	0.405513	0.405755	0.404844	0.393418	0.392934	0.393587	0.405459	0.405013
5	110	0.344875	0.345331	0.344477	0.334737	0.334044	0.334733	0.345122	0.344706
5	120	0.291460	0.292065	0.291301	0.282975	0.282252	0.282805	0.291919	0.291546
5	130	0.245152	0.245837	0.245183	0.238043	0.237392	0.237738	0.245735	0.245411
5	140	0.205490	0.206198	0.205657	0.199516	0.198986	0.199119	0.206125	0.205852
6	60	0.624048	0.623954	0.623528	0.122078	0.131483	0.122097	0.623730	0.623512
6	70	0.552563	0.552496	0.552075	0.109039	0.113071	0.109071	0.552298	0.552071
6	80	0.485882	0.485864	0.485457	0.096316	0.097051	0.096350	0.485694	0.485468
6	90	0.425220	0.425259	0.424875	0.084370	0.083236	0.084396	0.425117	0.424901
6	100	0.370967	0.371062	0.370705	0.073450	0.071392	0.073462	0.370943	0.370742
6	110	0.323013	0.323155	0.322829	0.063656	0.061276	0.063653	0.323057	0.322873
6	120	0.280971	0.281149	0.280856	0.054992	0.052652	0.054974	0.281068	0.280903
6	130	0.244318	0.244521	0.244260	0.047405	0.045308	0.047374	0.244455	0.244308
6	140	0.212483	0.212702	0.212471	0.040809	0.039054	0.040768	0.212646	0.212517
7	60	0.427492	0.427496	0.427439	0.000000	0.000000	0.000000	0.427490	0.427443
7	70	0.379755	0.379766	0.379714	0.000000	0.000000	0.000000	0.379760	0.379717
7	80	0.339300	0.339317	0.339269	0.000000	0.000000	0.000000	0.339310	0.339273
7	90	0.304747	0.304767	0.304725	0.000000	0.000000	0.000000	0.304761	0.304728
7	100	0.275015	0.275038	0.275000	0.000000	0.000000	0.000000	0.275032	0.275003
7	110	0.249255	0.249281	0.249246	0.000000	0.000000	0.000000	0.249275	0.249249
7	120	0.226796	0.226824	0.226793	0.000000	0.000000	0.000000	0.226818	0.226795

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Table 3 – continued from previous page

Set	K	Merton-0	Merton-$\hat{t}$	Merton-Higher	Kou-0	Kou-$\hat{t}$	Kou-Higher	Gaussian	Integral
7	130	0.207103	0.207132	0.207103	0.000000	0.000000	0.000000	0.207125	0.207106
7	140	0.189742	0.189772	0.189746	0.000000	0.000000	0.000000	0.189766	0.189748
8	60	0.695957	0.696648	0.694492	0.694825	0.694714	0.694713	0.694465	0.694130
8	70	0.625980	0.626167	0.624186	0.624334	0.624214	0.624701	0.624030	0.623598
8	80	0.554878	0.554801	0.552924	0.553021	0.552942	0.553658	0.552845	0.552331
8	90	0.485437	0.485439	0.722315	0.483743	0.483744	0.246879	0.483736	0.483162
8	100	0.419802	0.420127	0.418118	0.418532	0.418630	0.418992	0.418700	0.418094
8	110	0.359429	0.360158	0.358096	0.358673	0.358865	0.358793	0.358998	0.358385
8	120	0.305125	0.306205	0.304194	0.304841	0.305109	0.304558	0.305286	0.304687
8	130	0.257163	0.258474	0.256619	0.257239	0.257558	0.256559	0.257760	0.257193
8	140	0.215421	0.216832	0.215205	0.215729	0.216074	0.214709	0.216285	0.215763
9	60	0.764546	0.762835	0.762553	0.762874	0.762192	0.762120	0.762629	0.761800
9	70	0.691108	0.688958	0.688419	0.688768	0.688088	0.688923	0.688914	0.687701
9	80	0.608186	0.606520	0.605286	0.606038	0.605575	0.606783	0.606705	0.605212
9	90	0.522068	0.521469	0.519559	0.520719	0.520563	0.521548	0.521838	0.520234
9	100	0.438536	0.439007	0.436830	0.438084	0.438204	0.438593	0.439456	0.437912
9	110	0.361720	0.362917	0.360909	0.361935	0.362242	0.361953	0.363345	0.361989
9	120	0.293906	0.295438	0.293839	0.294487	0.294890	0.294045	0.295774	0.294676
9	130	0.235890	0.237469	0.236320	0.236601	0.237034	0.235838	0.237679	0.236854
9	140	0.187454	0.188920	0.188151	0.188157	0.188579	0.187239	0.189000	0.188431
10	60	0.452335	0.452335	0.841650	0.000000	0.000000	0.835138	0.452360	0.452299
10	70	0.401118	0.401121	0.783863	0.000000	0.000000	0.775459	0.401145	0.401088
10	80	0.357454	0.357459	0.691065	0.000000	0.000000	0.690034	0.357480	0.357428
10	90	0.320024	0.320030	0.578255	0.000000	0.000000	0.586608	0.320050	0.320002
10	100	0.287753	0.287761	0.468577	0.000000	0.000000	0.477414	0.287778	0.287735
10	110	0.259774	0.259782	0.369016	0.000000	0.000000	0.372834	0.259798	0.259758
10	120	0.235384	0.235392	0.279955	0.000000	0.000000	0.280603	0.235407	0.235370
10	130	0.214014	0.214023	0.205137	0.000000	0.000000	0.204804	0.214036	0.214003
10	140	0.195202	0.195211	0.146455	0.000000	0.000000	0.145927	0.195223	0.195192
11	60	0.162867	0.162867	0.790292			0.787936	0.162868	0.162866
11	70	0.147647	0.147647	0.721067			0.719045	0.147648	0.147646
11	80	0.135233	0.135233	0.635940			0.636332	0.135234	0.135232
11	90	0.124875	0.124875	0.543054			0.545779	0.124876	0.124875
11	100	0.116078	0.116079	0.451198			0.454433	0.116079	0.116078
11	110	0.108498	0.108499	0.366321			0.368511	0.108499	0.108498
11	120	0.101888	0.101888	0.291445			0.292211	0.101889	0.101888
11	130	0.096065	0.096065	0.227772			0.227503	0.096066	0.096065
11	140	0.090891	0.090891	0.175343			0.174578	0.090892	0.090890
12	60	0.863307	0.848999	0.862084	0.858305	0.855329	0.854890	0.860188	0.856503
12	70	0.806446	0.788662	0.798175	0.798604	0.795476	0.797352	0.805785	0.796484
12	80	0.712263	0.700080	0.697620	0.706760	0.704963	0.708388	0.717220	0.705583
12	90	0.594821	0.590563	0.584385	0.593439	0.592896	0.595000	0.602627	0.593180
12	100	0.473722	0.473521	0.470248	0.474298	0.474274	0.474809	0.480386	0.474367
12	110	0.361971	0.362818	0.361900	0.362809	0.362910	0.362732	0.366227	0.362918
12	120	0.266707	0.267601	0.267578	0.267362	0.267476	0.267085	0.268993	0.267453
12	130	0.190549	0.191372	0.191628	0.191068	0.191185	0.190762	0.191605	0.191154
12	140	0.132727	0.133565	0.133938	0.133218	0.133353	0.132947	0.133112	0.133320
13	60	0.791773	0.788443	0.790292	0.788636	0.788286	0.787936	0.787846	0.787986
13	70	0.724016	0.718728	0.721067	0.719168	0.718649	0.719045	0.719094	0.718424
13	80	0.640080	0.635623	0.635940	0.635922	0.635487	0.636332	0.637087	0.635340
13	90	0.547142	0.545299	0.543054	0.545199	0.545016	0.545779	0.547517	0.544914
13	100	0.453918	0.454555	0.451198	0.454069	0.454134	0.454433	0.456965	0.454049
13	110	0.367218	0.369244	0.366321	0.368533	0.368753	0.368511	0.371361	0.368673
13	120	0.290922	0.293343	0.291445	0.292565	0.292848	0.292211	0.294898	0.292770
13	130	0.226499	0.228782	0.227772	0.228035	0.228324	0.227503	0.229720	0.228251
13	140	0.173840	0.175815	0.175343	0.175136	0.175409	0.174578	0.176211	0.175342
14	60	0.828519	0.816185	0.828066	0.821422	0.821779	0.822124	0.819966	0.821898
14	70	0.767863	0.751587	0.764583	0.758058	0.758722	0.758392	0.759570	0.758633
14	80	0.679333	0.666814	0.671743	0.672541	0.673104	0.672280	0.678356	0.672931
14	90	0.572996	0.567930	0.564316	0.571710	0.571945	0.571198	0.580047	0.571808
14	100	0.464094	0.464489	0.459600	0.466307	0.466288	0.466049	0.473973	0.466224
14	110	0.363617	0.366214	0.364293	0.366643	0.366514	0.366762	0.371573	0.366508
14	120	0.276792	0.280078	0.280440	0.279667	0.279489	0.280007	0.281502	0.279521
14	130	0.205464	0.209253	0.210130	0.208338	0.208112	0.208711	0.207720	0.208167
14	140	0.149438	0.153777	0.153989	0.152537	0.152253	0.152801	0.150426	0.152322
15	60	0.841321	0.826136	0.841650	0.832854	0.833801	0.835138	0.831064	0.834198
15	70	0.786085	0.765531	0.783863	0.773943	0.776148	0.775459	0.776425	0.775701
15	80	0.699240	0.682832	0.691065	0.690542	0.692539	0.690034	0.699511	0.691789
15	90	0.589829	0.582927	0.578255	0.588345	0.589201	0.586608	0.601316	0.588639
15	100	0.475434	0.475505	0.468577	0.478395	0.478386	0.477414	0.490612	0.478117
15	110	0.369335	0.371563	0.369016	0.372631	0.372338	0.372834	0.381274	0.372274
15	120	0.277540	0.279678	0.279955	0.279787	0.279478	0.280603	0.284624	0.279517
15	130	0.202318	0.204158	0.205137	0.203885	0.203583	0.204804	0.205783	0.203662
15	140	0.143652	0.145494	0.146455	0.145123	0.144779	0.145927	0.145033	0.144873

Table 4: BCON Style Option Values: $S_{t_0} = 100, r = 0.05, q = 0.02, \sigma_{\mathrm{CGMY}} = 0.2, T = 0.5$

Set	K	Merton-0	Merton-$\hat{t}$	Merton-Higher	Kou-0	Kou-$\hat{t}$	Kou-Higher	Gaussian	Integral
1	60	0.961325	0.962123	0.961997	0.963711	0.956218	0.953038	0.962173	0.961040
1	70	0.954532	0.936377	0.954579	0.946888	0.922826	0.927346	0.941899	0.936987
1	80	0.914269	0.850463	0.894352	0.892607	0.852021	0.876786	0.891796	0.871737
1	90	0.751271	0.697225	0.656695	0.738517	0.715112	0.771180	0.766918	0.727345
1	100	0.502444	0.502984	0.473689	0.509069	0.509255	0.518663	0.536808	0.511654

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Set	K	Merton-0	Merton-$\hat{t}$	Merton-Higher	Kou-0	Kou-$\hat{t}$	Kou-Higher	Gaussian	Integral
1	110	0.293446	0.296781	0.301930	0.295153	0.296492	0.289357	0.302307	0.296239
1	120	0.140629	0.145015	0.148228	0.141800	0.143712	0.137733	0.141447	0.143131
1	130	0.055545	0.061681	0.064804	0.057092	0.060491	0.055745	0.055516	0.059788
1	140	0.018932	0.022375	0.023648	0.019873	0.023056	0.022882	0.019064	0.022589
2	60	0.930838	0.920023	0.927707	0.931600	0.925411	0.924679	0.930502	0.928349
2	70	0.880896	0.858057	0.874183	0.876336	0.867733	0.871551	0.874490	0.870625
2	80	0.789469	0.764572	0.776986	0.780988	0.773094	0.782490	0.780626	0.775021
2	90	0.649124	0.637912	0.634901	0.645262	0.641822	0.650961	0.648829	0.642533
2	100	0.487199	0.489116	0.480320	0.489084	0.489699	0.491798	0.494892	0.489587
2	110	0.337198	0.342511	0.338262	0.340113	0.342009	0.337883	0.344861	0.341596
2	120	0.216318	0.220534	0.219678	0.218178	0.219914	0.214297	0.220779	0.219486
2	130	0.129390	0.132437	0.132288	0.130409	0.131845	0.127632	0.131477	0.131472
2	140	0.073068	0.075506	0.075193	0.073649	0.074896	0.072651	0.073998	0.074569
3	60	0.937368	0.927238	0.934723	0.938423	0.931404	0.930769	0.937194	0.935140
3	70	0.893788	0.867225	0.887106	0.888522	0.877778	0.882405	0.886279	0.882112
3	80	0.805806	0.774211	0.791113	0.795919	0.785489	0.797161	0.795695	0.788910
3	90	0.659964	0.645408	0.641922	0.656063	0.651505	0.662983	0.661288	0.653167
3	100	0.488462	0.490337	0.480540	0.491178	0.491785	0.494203	0.498863	0.492073
3	110	0.331013	0.336345	0.333005	0.334167	0.336090	0.331505	0.339711	0.335810
3	120	0.206188	0.210205	0.210019	0.207939	0.209631	0.204135	0.210574	0.209242
3	130	0.118956	0.121978	0.121922	0.119881	0.121331	0.117707	0.120811	0.120959
3	140	0.064660	0.067211	0.066740	0.065184	0.066488	0.064896	0.065453	0.066144
4	60	0.960406	0.965446	0.933374	0.961430	0.959982	0.966817	0.960394	0.952631
4	70	0.924765	0.909989	0.884431	0.921301	0.902605	0.929729	0.919789	0.906003
4	80	0.825977	0.792985	0.782556	0.818801	0.799547	0.835025	0.822322	0.807048
4	90	0.658627	0.645978	0.635004	0.656179	0.650543	0.668033	0.663860	0.653450
4	100	0.473028	0.473927	0.469606	0.474394	0.474769	0.476005	0.479615	0.475234
4	110	0.309283	0.313758	0.314071	0.311251	0.313247	0.308164	0.312891	0.312864
4	120	0.185832	0.192649	0.193220	0.187739	0.191113	0.184714	0.187696	0.190165
4	130	0.104533	0.113194	0.112724	0.106045	0.110808	0.105089	0.105793	0.109293
4	140	0.056368	0.064632	0.064161	0.057262	0.062600	0.058395	0.057258	0.060773
5	60	0.919651	0.911144	0.918556	0.920290	0.918658	0.915469	0.918808	0.918447
5	70	0.868017	0.848862	0.864478	0.861589	0.857930	0.858072	0.858950	0.857874
5	80	0.777101	0.754716	0.768242	0.766919	0.762360	0.767519	0.765088	0.762615
5	90	0.639463	0.629075	0.627301	0.635690	0.633348	0.639471	0.637615	0.633655
5	100	0.481934	0.484424	0.475932	0.485100	0.485732	0.486868	0.489895	0.485771
5	110	0.336860	0.343394	0.339422	0.341201	0.343100	0.339138	0.345505	0.342847
5	120	0.220133	0.226312	0.225352	0.223232	0.225278	0.220205	0.225686	0.224859
5	130	0.135955	0.141294	0.140181	0.137924	0.139867	0.136405	0.139107	0.139393
5	140	0.080873	0.085365	0.083554	0.082031	0.083745	0.082377	0.082694	0.083294
6	60	0.883749	0.879247	0.881406	0.882886	0.881381	0.880007	0.882008	0.881003
6	70	0.808162	0.801835	0.804131	0.804833	0.802868	0.803989	0.803872	0.802374
6	80	0.705351	0.699731	0.700082	0.701327	0.699700	0.702633	0.701046	0.699185
6	90	0.584019	0.581926	0.579324	0.581942	0.581338	0.583846	0.582798	0.580863
6	100	0.459983	0.461359	0.457623	0.460241	0.460657	0.461050	0.461935	0.460247
6	110	0.347052	0.350085	0.347176	0.348483	0.349478	0.347748	0.350371	0.349139
6	120	0.252683	0.255872	0.254232	0.254247	0.255402	0.252622	0.255871	0.255132
6	130	0.178768	0.181473	0.180682	0.180026	0.181118	0.178386	0.181243	0.180906
6	140	0.123730	0.125842	0.125444	0.124615	0.125563	0.123503	0.125470	0.125400
7	60	0.723005	0.722483	0.721877	0.605333	0.602449	0.605483	0.722555	0.721959
7	70	0.633109	0.632751	0.632153	0.529835	0.528192	0.530164	0.632802	0.632259
7	80	0.545939	0.545804	0.545248	0.456168	0.455575	0.456475	0.545831	0.545376
7	90	0.465428	0.465513	0.465024	0.387902	0.388287	0.388072	0.465513	0.465158
7	100	0.393542	0.393808	0.393399	0.326869	0.328148	0.326872	0.393780	0.393525
7	110	0.330869	0.331263	0.330937	0.273668	0.275716	0.273526	0.331210	0.331045
7	120	0.277141	0.277614	0.277365	0.228124	0.230790	0.227883	0.277540	0.277450
7	130	0.231626	0.232136	0.231955	0.189631	0.192758	0.189343	0.232046	0.232018
7	140	0.193386	0.193903	0.193780	0.157392	0.160833	0.157100	0.193802	0.193822
8	60	0.909727	0.898897	0.908660	0.911325	0.909817	0.905429	0.909635	0.909021
8	70	0.865838	0.839043	0.862516	0.855957	0.852501	0.851713	0.852615	0.851309
8	80	0.789212	0.754781	0.779342	0.772179	0.767024	0.772722	0.768294	0.765767
8	90	0.662788	0.644590	0.644118	0.655477	0.652101	0.661282	0.655258	0.650999
8	100	0.507272	0.512370	0.491368	0.513354	0.514529	0.518058	0.519247	0.513520
8	110	0.359103	0.371501	0.358577	0.367039	0.370634	0.363647	0.375322	0.369658
8	120	0.234609	0.242696	0.241069	0.238236	0.241221	0.229248	0.244225	0.240378
8	130	0.138580	0.142886	0.143753	0.139710	0.141673	0.131449	0.142431	0.141022
8	140	0.073349	0.076690	0.077940	0.073931	0.075693	0.068877	0.074590	0.075168
9	60	0.943385	0.934237	0.943168	0.945593	0.940071	0.938064	0.943886	0.943419
9	70	0.915515	0.881568	0.913116	0.907478	0.896264	0.898903	0.903421	0.901494
9	80	0.845852	0.794134	0.832460	0.829298	0.814350	0.826915	0.826754	0.820472
9	90	0.694825	0.665930	0.664067	0.689293	0.681144	0.698474	0.697632	0.685404
9	100	0.498196	0.500458	0.483502	0.504855	0.505509	0.509373	0.518646	0.506811
9	110	0.319180	0.327300	0.328084	0.324157	0.326770	0.318838	0.330877	0.326437
9	120	0.179182	0.188776	0.191358	0.182507	0.186021	0.177060	0.182565	0.184971
9	130	0.088643	0.099957	0.098032	0.091537	0.096438	0.091161	0.090026	0.094913
9	140	0.040948	0.050009	0.045526	0.042683	0.047544	0.046801	0.042006	0.045838
10	60	0.744233	0.743858	0.743377	105.016527	883.145346	105.774553	0.744336	0.743641
10	70	0.651768	0.651566	0.651117	-9.931039	347.304985	-10.080893	0.651937	0.651321
10	80	0.560889	0.560816	0.560422	24.044312	72.577570	24.395858	0.561105	0.560579
10	90	0.476340	0.476356	0.476022	-1.425523	24.147696	-1.440147	0.476581	0.476142
10	100	0.400591	0.400664	0.400387	-3.435720	-0.427779	-3.449349	0.400838	0.400479
10	110	0.334499	0.334603	0.334378	8.561858	2.252761	8.543299	0.334738	0.334447
10	120	0.277908	0.278025	0.277845	4.798560	0.282456	4.759237	0.278129	0.277896
10	130	0.230100	0.230219	0.230075	3.674488	0.035018	3.623842	0.230299	0.230114
10	140	0.190099	0.190214	0.190100	2.464988	0.018732	2.418608	0.190275	0.190128
11	60	0.438816	0.438817	0.438800	0.000000	0.000000	0.000000	0.438828	0.438801
11	70	0.388417	0.388419	0.388403	0.000000	0.000000	0.000000	0.388429	0.388404
11	80	0.346044	0.346047	0.346032	0.000000	0.000000	0.000000	0.346056	0.346033
11	90	0.310085	0.310089	0.310075	0.000000	0.000000	0.000000	0.310097	0.310076

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Table 4 – continued from previous page

Set	K	Merton-0	Merton-$\hat{t}$	Merton-Higher	Kou-0	Kou-$\hat{t}$	Kou-Higher	Gaussian	Integral
11	100	0.279309	0.279312	0.279300	0.000000	0.000000	0.000000	0.279320	0.279301
11	110	0.252763	0.252767	0.252756	0.000000	0.000000	0.000000	0.252773	0.252756
11	120	0.229705	0.229709	0.229699	0.000000	0.000000	0.000000	0.229715	0.229700
11	130	0.209551	0.209555	0.209545	0.000000	0.000000	0.000000	0.209560	0.209546
11	140	0.191832	0.191836	0.191827	0.000000	0.000000	0.000000	0.191841	0.191828
12	60	0.969851	0.968037	0.971043	0.970392	0.965247	0.967430	0.968071	0.970059
12	70	0.968660	0.954808	0.976840	0.961371	0.947040	0.949455	0.956711	0.958643
12	80	0.945418	0.901205	0.974946	0.917779	0.893889	0.884594	0.927196	0.906725
12	90	0.797137	0.741392	0.713220	0.763851	0.748891	0.773724	0.822361	0.753112
12	100	0.502116	0.501805	0.476357	0.505417	0.505369	0.509230	0.529743	0.505903
12	110	0.265495	0.269561	0.284083	0.267485	0.268207	0.263415	0.265006	0.267990
12	120	0.098784	0.114233	0.146403	0.109482	0.114787	0.102217	0.095694	0.114483
12	130	0.026315	0.031979	0.025616	0.035492	0.041510	0.049507	0.026684	0.041358
12	140	0.007807	0.009064	0.003981	0.011070	0.014976	0.020754	0.008928	0.013479
13	60	0.948389	0.937926	0.949237	0.949935	0.950380	0.946950	0.947185	0.949392
13	70	0.932081	0.893662	0.933638	0.920218	0.917259	0.915766	0.913760	0.917836
13	80	0.877821	0.814164	0.875125	0.855074	0.846653	0.851200	0.850344	0.850516
13	90	0.725024	0.684540	0.690349	0.717669	0.711655	0.722008	0.732857	0.715533
13	100	0.502634	0.504327	0.480499	0.513338	0.513592	0.516393	0.537708	0.514793
13	110	0.303085	0.311251	0.320402	0.308383	0.309721	0.304517	0.314348	0.309464
13	120	0.150612	0.165146	0.165019	0.156874	0.159584	0.155233	0.151818	0.158733
13	130	0.065274	0.080429	0.069347	0.070860	0.074271	0.073937	0.067060	0.073080
13	140	0.028289	0.038197	0.028216	0.030843	0.033438	0.034649	0.030190	0.032227
14	60	0.959873	0.944315	0.960588	0.958434	0.957247	0.959575	0.953417	0.958624
14	70	0.953592	0.907549	0.957859	0.939196	0.939167	0.941582	0.929094	0.940190
14	80	0.913380	0.836456	0.928417	0.883669	0.889940	0.888317	0.882667	0.886081
14	90	0.759359	0.703650	0.726279	0.740077	0.744913	0.738533	0.783523	0.741763
14	100	0.500139	0.500713	0.468999	0.510114	0.510072	0.508262	0.549855	0.509555
14	110	0.271180	0.287946	0.321077	0.281972	0.280611	0.285848	0.263155	0.281060
14	120	0.106603	0.140876	0.109993	0.131617	0.127624	0.127901	0.103883	0.128946
14	130	0.039594	0.067084	0.031445	0.054987	0.051053	0.049427	0.046261	0.053167
14	140	0.016804	0.033906	0.012353	0.022749	0.020476	0.019570	0.023598	0.022000
15	60	0.962917	0.945720	0.963498	0.960679		0.963381	0.954751	0.961124
15	70	0.958657	0.911465	0.963426	0.944206	0.941586	0.950473	0.932938	0.946533
15	80	0.924159	0.844837	0.947400	0.891221	0.908002	0.907069	0.892470	0.897581
15	90	0.777322	0.715460	0.762863	0.748682	0.763967	0.748373	0.806116	0.754377
15	100	0.506457	0.506544	0.459976	0.517263	0.517246	0.510403	0.572618	0.515819
15	110	0.274859	0.281362	0.310511	0.279685	0.278139	0.288696	0.275180	0.278614
15	120	0.101881	0.124152	0.116610	0.128921	0.120358	0.121414	0.096098	0.121504
15	130	0.032592	0.047976	0.024311	0.050453	0.041967	0.030870	0.033400	0.045108
15	140	0.011514	0.019556	0.007238	0.017237	0.012094	0.007158	0.014013	0.015451

Table 5: Vanilla Call Option Values: $S_{t_0} = 100, r = 0.05, q = 0.02, \sigma_{\mathrm{CGMY}} = 0.2, T = 1$

Set	K	Merton-0	Merton-$\hat{t}$	Merton-Higher	Kou-0	Kou-$\hat{t}$	Kou-Higher	Gaussian	Integral	FFT
1	60	41.11396	41.57192	42.00354	41.25490	41.35146	41.04554	41.28881	41.31072	41.31068
1	70	32.01508	32.74138	33.54328	32.33608	32.44855	32.07189	32.33379	32.39618	32.39587
1	80	24.12423	24.41252	25.08618	24.23593	24.20920	23.87412	24.04184	24.17028	24.17041
1	90	17.56998	17.08802	17.53667	17.23499	17.05010	16.82799	16.87805	17.04633	17.04700
1	100	11.85143	11.28716	11.56581	11.48637	11.32703	11.21062	11.19886	11.34392	11.34494
1	110	7.32694	7.08511	7.23784	7.19946	7.12469	7.08262	7.04798	7.14378	7.14459
1	120	4.30274	4.24558	4.32418	4.29946	4.26973	4.26498	4.22745	4.28400	4.28495
1	130	2.45056	2.44450	2.48605	2.47197	2.45744	2.46451	2.43286	2.46642	2.46728
1	140	1.35994	1.36407	1.38830	1.37997	1.37030	1.37756	1.35371	1.37537	1.37579
2	60	42.16669	42.19688	42.24730	42.20662	42.21499	42.15241	42.21420	42.22537	42.22534
2	70	34.06541	34.11608	34.17307	34.15044	34.14585	34.07888	34.14301	34.15986	34.15974
2	80	26.92802	26.92027	26.97875	26.98457	26.95411	26.89402	26.94912	26.97079	26.97096
2	90	20.86747	20.75006	20.80558	20.83828	20.78182	20.73630	20.77510	20.79963	20.80004
2	100	15.84477	15.65054	15.69991	15.74682	15.67717	15.64887	15.66942	15.69449	15.69506
2	110	11.78069	11.57771	11.62141	11.66639	11.59856	11.58393	11.59053	11.61421	11.61468
2	120	8.58794	8.42293	8.45661	8.49538	8.43866	8.43806	8.43095	8.45202	8.45263
2	130	6.15734	6.04325	6.06978	6.09788	6.05491	6.06182	6.04789	6.06583	6.06649
2	140	4.35830	4.28791	4.30848	4.32711	4.29652	4.30710	4.29037	4.30518	4.30557
3	60	41.96007	42.01521	42.08237	42.01409	42.02337	41.95157	42.02085	42.03328	42.03325
3	70	33.71374	33.79081	33.86590	33.82290	33.81755	33.73943	33.81132	33.83132	33.83118
3	80	26.45971	26.44415	26.51991	26.51916	26.48075	26.41029	26.47084	26.49764	26.49782
3	90	20.31325	20.15069	20.22115	20.25753	20.18754	20.13423	20.17500	20.20590	20.20635
3	100	15.22173	14.97864	15.03966	15.09194	15.01008	14.97701	14.99646	15.02798	15.02860
3	110	11.11914	10.89019	10.93748	10.98879	10.91464	10.90206	10.90140	10.93063	10.93114
3	120	7.93567	7.76894	7.80786	7.84437	7.78706	7.78533	7.77517	7.80045	7.80111
3	130	5.55941	5.45662	5.48622	5.51003	5.46979	5.47588	5.45968	5.48048	5.48117
3	140	3.84214	3.78599	3.80822	3.82229	3.79555	3.80500	3.78725	3.80381	3.80421
4	60	41.52335	41.67429	41.81373	41.56557	41.61597	41.48262	41.55791	41.59227	41.59223
4	70	33.07958	33.16096	33.27177	33.13931	33.14098	33.03298	33.09165	33.13524	33.13511
4	80	25.63239	25.58405	25.66988	25.64238	25.59251	25.51429	25.55397	25.59891	25.59917
4	90	19.32943	19.18985	19.25582	19.27874	19.20591	19.15502	19.17658	19.21764	19.21821
4	100	14.20370	14.04512	14.09596	14.13147	14.06047	14.02976	14.03786	14.07339	14.07411
4	110	10.20891	10.07610	10.11682	10.14892	10.08935	10.06995	10.07108	10.10158	10.10213
4	120	7.21367	7.11975	7.15447	7.17835	7.13131	7.11565	7.11553	7.14214	7.14282
4	130	5.03547	4.97805	5.00960	5.02384	4.98834	4.97144	4.97376	4.99751	4.99818
4	140	3.48700	3.45879	3.48880	3.49265	3.46778	3.44720	3.45364	3.47518	3.47556
5	60	42.45414	42.48078	42.51618	42.51262	42.51520	42.47628	42.52006	42.52527	42.52525
5	70	34.50807	34.56799	34.61323	34.61643	34.60942	34.56571	34.61291	34.62241	34.62230
5	80	27.55079	27.53354	27.58506	27.60506	27.57707	27.53595	27.57738	27.59210	27.59225

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Table 5 – continued from previous page

Set	K	Merton-0	Merton-$\hat{t}$	Merton-Higher	Kou-0	Kou-$\hat{t}$	Kou-Higher	Gaussian	Integral	FFT
5	90	21.65096	21.49300	21.54687	21.58606	21.53454	21.50148	21.53064	21.55048	21.55086
5	100	16.73106	16.47743	16.53005	16.58047	16.51439	16.49175	16.50643	16.53013	16.53066
5	110	12.70259	12.43948	12.48816	12.53858	12.47091	12.45828	12.45997	12.48558	12.48601
5	120	9.48957	9.27529	9.31855	9.36118	9.30135	9.29661	9.28893	9.31442	9.31499
5	130	6.99997	6.85137	6.88877	6.92077	6.87279	6.87320	6.86027	6.88406	6.88467
5	140	5.11904	5.02810	5.05989	5.08169	5.04573	5.04872	5.03412	5.05522	5.05559
6	60	43.66785	43.67316	43.68881	41.62386	42.09126	42.06744	43.68695	43.69199	43.69198
6	70	36.41446	36.41526	36.43410	34.70273	35.01844	34.99470	36.42961	36.43666	36.43661
6	80	30.05802	30.03704	30.05788	28.62500	28.82643	28.80567	30.05083	30.05964	30.05977
6	90	24.60436	24.55240	24.57407	23.40097	23.52098	23.50485	24.56489	24.57501	24.57529
6	100	20.00143	19.92253	19.94399	18.99140	19.05697	19.04605	19.93338	19.94429	19.94465
6	110	16.16734	16.07367	16.09421	15.32493	15.35664	15.35055	16.08281	16.09400	16.09429
6	120	13.00899	12.91355	12.93269	12.31353	12.32594	12.32386	12.92111	12.93214	12.93254
6	130	10.43198	10.34432	10.36180	9.86435	9.86697	9.86790	10.35051	10.36105	10.36150
6	140	8.34607	8.27130	8.28705	7.88764	7.88623	7.88918	8.27637	8.28621	8.28649
7	60	49.78649	49.77944	49.78308	1.22341	1.20736	1.20717	49.78049	49.78292	49.78291
7	70	44.37279	44.36306	44.36717	1.10126	1.09100	1.09084	44.36417	44.36698	44.36694
7	80	39.62117	39.60899	39.61346	0.99680	0.98271	0.98258	39.61013	39.61324	39.61327
7	90	35.45232	35.43808	35.44280	0.90647	0.88411	0.88400	35.43923	35.44257	35.44264
7	100	31.79226	31.77641	31.78132	0.82748	0.79541	0.79532	31.77756	31.78107	31.78117
7	110	28.57453	28.55753	28.56254	0.75772	0.71614	0.71607	28.55866	28.56229	28.56237
7	120	25.74060	25.72287	25.72794	0.69559	0.64555	0.64549	25.72399	25.72768	25.72781
7	130	23.23960	23.22150	23.22659	0.63987	0.58278	0.58274	23.22260	23.22632	23.22647
7	140	21.02755	21.00940	21.01428	0.58961	0.52698	0.52696	21.01047	21.01420	21.01430
8	60	42.67437	42.71202	42.77031	42.77330	42.78017	42.71442	42.79077	42.79674	42.79672
8	70	34.61559	34.82684	34.89916	34.90746	34.91281	34.83743	34.92491	34.93475	34.93459
8	80	27.59845	27.75182	27.83234	27.85997	27.84556	27.77241	27.85725	27.87162	27.87169
8	90	21.75684	21.59701	21.67961	21.74149	21.68795	21.62819	21.69736	21.71629	21.71660
8	100	16.88093	16.41173	16.49109	16.58620	16.49163	16.45254	16.49745	16.52020	16.52068
8	110	12.75333	12.18220	12.25416	12.36304	12.24677	12.23064	12.24855	12.27366	12.27409
8	120	9.31563	8.84020	8.90205	9.00087	8.88897	8.89360	8.88712	8.91276	8.91337
8	130	6.58412	6.27909	6.32971	6.40419	6.31409	6.33445	6.30960	6.33403	6.33471
8	140	4.53050	4.37233	4.41213	4.46041	4.39657	4.42648	4.39060	4.41255	4.41297
9	60	41.64904	41.78802	41.90598	41.75931	41.77226	41.68961	41.77799	41.78651	41.78647
9	70	33.06040	33.30312	33.44799	33.31017	33.31555	33.21268	33.31405	33.33312	33.33292
9	80	25.60119	25.61511	25.76773	25.70441	25.65894	25.55713	25.64463	25.67930	25.67944
9	90	19.35954	18.98805	19.13249	19.15740	19.04988	18.96834	19.02198	19.07251	19.07301
9	100	14.10336	13.58225	13.70783	13.77072	13.64305	13.59021	13.60596	13.66596	13.66670
9	110	9.81057	9.40534	9.50695	9.56032	9.45431	9.42805	9.41431	9.47505	9.47566
9	120	6.56415	6.33438	6.41357	6.44448	6.37011	6.36201	6.33177	6.38720	6.38798
9	130	4.27992	4.17302	4.23506	4.24831	4.19783	4.19826	4.16308	4.21090	4.21168
9	140	2.74331	2.70675	2.75688	2.75728	2.72281	2.72450	2.69213	2.73214	2.73257
10	60	48.66929	48.66687	48.66857	-0.09021	0.18278	0.18307	48.66683	48.66831	48.66830
10	70	43.02971	43.02661	43.02839	-0.22453	0.08918	0.08841	43.02657	43.02814	43.02811
10	80	38.08945	38.08579	38.08760	-0.04339	0.10741	0.10758	38.08576	38.08736	38.08740
10	90	33.76961	33.76553	33.76732	-0.06099	0.07204	0.07219	33.76550	33.76710	33.76719
10	100	29.99383	29.98945	29.99120	-0.01969	0.04360	0.04373	29.98943	29.99099	29.99111
10	110	26.69192	26.68735	26.68903	-0.07310	-0.04741	-0.04760	26.68733	26.68885	26.68895
10	120	23.80119	23.79654	23.79815	0.00104	0.03854	0.03864	23.79652	23.79798	23.79812
10	130	21.26665	21.26199	21.26351	0.01798	0.00778	0.00780	21.26198	21.26336	21.26353
10	140	19.04051	19.03590	19.03728	-0.00722	-0.00139	-0.00139	19.03589	19.03720	19.03732
11	60	68.05364	68.05350	68.05356	0.00000	0.00000	0.00000	68.05350	68.05356	68.05355
11	70	65.24653	65.24638	65.24644	0.00000	0.00000	0.00000	65.24638	65.24644	65.24638
11	80	62.71791	62.71774	62.71781	0.00000	0.00000	0.00000	62.71774	62.71781	62.71775
11	90	60.42086	60.42068	60.42075	0.00000	0.00000	0.00000	60.42068	60.42075	60.42070
11	100	58.31955	58.31936	58.31943	0.00000	0.00000	0.00000	58.31936	58.31943	58.31940
11	110	56.38594	56.38574	56.38582	0.00000	0.00000	0.00000	56.38574	56.38582	56.38580
11	120	54.59763	54.59742	54.59750	0.00000	0.00000	0.00000	54.59742	54.59750	54.59748
11	130	52.93643	52.93621	52.93629	0.00000	0.00000	0.00000	52.93621	52.93629	52.93627
11	140	51.38734	51.38711	51.38720	0.00000	0.00000	0.00000	51.38711	51.38719	51.38719
12	60	40.96668	41.25611	41.39210	41.06439	41.08305	41.00264	41.08787	41.08034	41.08029
12	70	31.63715	32.25378	33.12806	31.85608	31.90278	31.73387	31.84071	31.87418	31.87376
12	80	23.20826	23.65119	24.72678	23.30112	23.29289	23.09708	23.05238	23.25005	23.25018
12	90	16.20066	15.92216	16.46499	15.87346	15.79189	15.68866	15.47588	15.77676	15.77760
12	100	10.19505	9.91899	10.18515	9.96522	9.92865	9.88322	9.74718	9.93023	9.93150
12	110	5.87658	5.79722	5.91188	5.83220	5.82015	5.80484	5.73429	5.82404	5.82499
12	120	3.23688	3.20113	3.26499	3.22220	3.21139	3.20512	3.15313	3.21378	3.21480
12	130	1.70622	1.69792	1.75389	1.70501	1.68962	1.68275	1.62615	1.68955	1.69039
12	140	0.84233	0.89474	0.96140	0.87547	0.86174	0.84985	0.78794	0.85809	0.85846
13	60	41.38928	41.59490	41.70613	41.59667	41.60027	41.56993	41.62008	41.61218	41.61214
13	70	32.56957	32.92489	33.10259	32.93087	32.93191	32.88289	32.95454	32.94636	32.94608
13	80	24.97741	24.95844	25.18312	25.01417	24.98140	24.92142	24.98116	24.99375	24.99385
13	90	18.62850	18.01390	18.24671	18.14891	18.07114	18.01648	18.02142	18.08207	18.08264
13	100	13.10412	12.37932	12.59042	12.54055	12.45813	12.41984	12.36428	12.46960	12.47050
13	110	8.57140	8.14776	8.31735	8.27640	8.22030	8.19974	8.11469	8.23143	8.23217
13	120	5.35965	5.17976	5.30676	5.26898	5.23348	5.22522	5.14036	5.24278	5.24367
13	130	3.27770	3.21405	3.30967	3.27400	3.24788	3.24456	3.17425	3.25463	3.25547
13	140	1.96421	1.97073	2.04581	2.00542	1.98640	1.98288	1.93227	1.99064	1.99107
14	60	41.09512	41.42518	41.60027	41.39121	41.39056	41.40159	41.41613	41.38194	41.38189
14	70	32.08722	32.56142	32.91336	32.44188	32.44508	32.47078	32.49310	32.43531	32.43494
14	80	24.24888	24.28792	24.81925	24.07592	24.10322	24.14832	24.10100	24.10157	24.10166
14	90	17.69181	16.94351	17.48738	16.79264	16.84784	16.89403	16.65792	16.84907	16.84980
14	100	11.78460	11.03915	11.49552	11.04538	11.08655	11.11917	10.75544	11.08371	11.08483
14	110	7.22228	6.84866	7.20460	6.89996	6.92433	6.94527	6.60583	6.92005	6.92092
14	120	4.41447	4.14627	4.44955	4.13658	4.16589	4.18319	3.89989	4.16354	4.16452
14	130	2.57439	2.53044	2.83625	2.43060	2.46012	2.48087	2.27580	2.46106	2.46189
14	140	1.37430	1.58894	1.90911	1.44040	1.45637	1.48067	1.36600	1.45870	1.45908
15	60	41.04622	41.37736	41.56079	41.34618	41.34598	41.37068	41.36940	41.32177	41.32172
15	70	31.97023	32.45830	32.86335	32.30166	32.31608	32.37576	32.37398	32.28540	32.28499

Continued on next page

Table 5 – continued from previous page

Set	K	Merton-0	Merton-$\hat{t}$	Merton-Higher	Kou-0	Kou-$\hat{t}$	Kou-Higher	Gaussian	Integral	FFT
15	80	23.97272	24.09231	24.77391	23.72996	23.81008	23.94254	23.83616	23.80885	23.80891
15	90	17.29812	16.58564	17.29944	16.22103	16.38164	16.53282	16.16665	16.39477	16.39552
15	100	11.17734	10.49064	11.04720	10.40593	10.50852	10.60042	10.11749	10.50495	10.50614
15	110	6.36961	6.21954	6.55821	6.26974	6.30234	6.35147	6.00992	6.29087	6.29180
15	120	3.59275	3.51559	3.70899	3.53336	3.56726	3.58536	3.37183	3.55787	3.55890
15	130	1.98220	1.93434	2.07295	1.87257	1.93159	1.94251	1.77680	1.92798	1.92885
15	140	1.01254	1.07362	1.21570	0.95942	1.01535	1.03357	0.90807	1.01827	1.01866

Table 6: Vanilla Call Option Values: $S_{t_0} = 100, r = 0.05, q = 0.02, \sigma_{\text{CGMY}} = 0.2, T = 2$

Set	K	Merton-0	Merton-$\hat{t}$	Merton-Higher	Kou-0	Kou-$\hat{t}$	Kou-Higher	Gaussian	Integral	FFT
1	60	42.66276	42.82800	42.96305	42.77141	42.78591	42.69467	42.76566	42.78722	42.78719
1	70	34.81573	34.92645	35.06687	34.92883	34.91326	34.81698	34.87687	34.91669	34.91652
1	80	27.87413	27.80406	27.93607	27.87808	27.82000	27.73465	27.77199	27.82770	27.82780
1	90	21.86856	21.63727	21.75340	21.75030	21.66864	21.60287	21.61737	21.68004	21.68040
1	100	16.77241	16.49638	16.59189	16.60844	16.52978	16.48596	16.48257	16.54275	16.54327
1	110	12.58119	12.35217	12.42612	12.44225	12.38079	12.35677	12.34153	12.39337	12.39381
1	120	9.26185	9.10733	9.16214	9.17212	9.12946	9.12056	9.09893	9.14052	9.14111
1	130	6.72055	6.62908	6.66860	6.67318	6.64532	6.64630	6.62249	6.65441	6.65505
1	140	4.82478	4.77521	4.80330	4.80464	4.78682	4.79322	4.77009	4.79397	4.79436
2	60	44.81778	44.82021	44.83309	42.05811	43.64720	43.62559	44.82921	44.83426	44.83424
2	70	38.04320	38.04094	38.05596	35.66467	36.92140	36.90009	38.05012	38.05671	38.05664
2	80	32.05658	32.04235	32.05860	30.01679	30.99411	30.97537	32.05111	32.05893	32.05900
2	90	26.85107	26.82094	26.83758	25.10681	25.85666	25.84190	26.82891	26.83755	26.83773
2	100	22.38257	22.33736	22.35370	20.89462	21.46388	21.45368	22.34434	22.35339	22.35365
2	110	18.58583	18.52978	18.54534	17.32009	17.74893	17.74324	18.53573	18.54484	18.54506
2	120	15.38644	15.32504	15.33951	14.31330	14.63478	14.63311	15.32999	15.33888	15.33920
2	130	12.70851	12.64679	12.65997	11.80190	12.04234	12.04404	12.65083	12.65930	12.65967
2	140	10.47930	10.42100	10.43285	9.71597	9.89584	9.90015	10.42425	10.43217	10.43241
3	60	44.42853	44.43246	44.44875	43.15798	43.63186	43.60713	44.44386	44.44991	44.44989
3	70	37.49873	37.49610	37.51511	36.41915	36.77012	36.74539	37.50756	37.51569	37.51561
3	80	31.37384	31.35366	31.37415	30.45446	30.70604	30.68411	31.36431	31.37413	31.37420
3	90	26.05850	26.01608	26.03689	25.27221	25.44734	25.42995	26.02543	26.03636	26.03657
3	100	21.51281	21.45126	21.47147	20.83982	20.95870	20.94659	21.45912	21.47058	21.47087
3	110	17.67149	17.59848	17.61745	17.09789	17.17704	17.17011	17.60488	17.61634	17.61659
3	120	14.45756	14.38145	14.42299	13.97247	14.02454	13.99860	14.38651	14.39758	14.39793
3	130	11.79097	11.71833	11.73386	11.38450	11.41868	11.42011	11.72225	11.73266	11.73306
3	140	9.59383	9.52877	9.54247	9.25630	9.27898	9.28324	9.53176	9.54132	9.54159
4	60	43.68416	43.68378	43.70286	43.63657	43.41363	43.38761	43.68065	43.69224	43.69222
4	70	36.51210	36.49545	36.51529	36.47259	36.31741	36.29353	36.49351	36.50703	36.50697
4	80	30.20034	30.16632	30.18581	30.16248	30.04721	30.02740	30.16483	30.17920	30.17934
4	90	24.77064	24.72374	24.74217	24.73321	24.64312	24.62794	24.72241	24.73681	24.73708
4	100	20.18577	20.13266	20.14971	20.15066	20.07768	20.06669	20.13141	20.14532	20.14567
4	110	16.37149	16.31814	16.33375	16.34113	16.28051	16.27277	16.31696	16.33009	16.33037
4	120	13.23498	13.18547	13.19972	13.21104	13.15973	13.15416	13.18437	13.19660	13.19699
4	130	10.67848	10.63502	10.64802	10.66156	10.61752	10.61319	10.63401	10.64531	10.64574
4	140	8.60819	8.57158	8.58349	8.59803	8.55983	8.55596	8.57065	8.58106	8.58134
5	60	45.37172	45.37568	45.38700	43.53126	44.18573	44.17073	45.38676	45.38985	45.38984
5	70	38.79094	38.78911	38.80295	37.20725	37.68944	37.67445	38.80103	38.80553	38.80546
5	80	32.98095	32.96383	32.97951	31.61515	31.96650	31.95303	32.97578	32.98166	32.98171
5	90	27.92137	27.88370	28.09150	26.73876	26.99293	26.78974	27.89505	27.90215	27.90232
5	100	23.56097	23.50365	23.52096	22.53434	22.71789	22.70984	23.51401	23.52208	23.52231
5	110	19.83310	19.76146	19.77871	18.94196	19.07526	19.07016	19.77063	19.77937	19.77957
5	120	16.66602	16.58703	16.60380	15.89437	15.99257	15.99019	16.59497	16.60408	16.60437
5	130	13.98890	13.90901	13.92501	13.32324	13.39732	13.39724	13.91578	13.92500	13.92533
5	140	11.73502	11.65911	11.67415	11.16314	11.22085	11.22262	11.66482	11.67392	11.67414
6	60	47.57578	47.57312	47.57814	7.59826	9.22918	9.22739	47.57625	47.57841	47.57839
6	70	41.70391	41.69764	41.70351	6.61544	7.88462	7.88298	41.70105	41.70373	41.70368
6	80	36.52486	36.51408	36.52054	5.76341	6.75535	6.75395	36.51759	36.52072	36.52075
6	90	31.98269	31.96717	31.97403	5.02748	5.80463	5.80353	31.97067	31.97416	31.97425
6	100	28.01405	27.99415	28.00121	4.39277	5.00220	5.00142	27.99755	28.00129	28.00143
6	110	24.55469	24.53117	24.53829	3.84537	4.32312	4.32264	24.53442	24.53833	24.53845
6	120	21.54328	21.51710	21.52416	3.37282	3.74684	3.74663	21.52016	21.52415	21.52433
6	130	18.92333	18.89547	18.90238	2.96427	3.25640	3.25644	18.89833	18.90234	18.90256
6	140	16.64399	16.61535	16.62205	2.61041	2.83786	2.83810	16.61801	16.62199	16.62213
7	60	56.98352	56.98065	56.98179	0.00000	0.00000	0.00000	56.98095	56.98177	56.98176
7	70	52.95490	52.95146	52.95274	0.00000	0.00000	0.00000	52.95178	52.95270	52.95264
7	80	49.36578	49.36185	49.36323	0.00000	0.00000	0.00000	49.36219	49.36319	49.36315
7	90	46.15055	46.14619	46.14766	0.00000	0.00000	0.00000	46.14654	46.14762	46.14760
7	100	43.25583	43.25110	43.25264	0.00000	0.00000	0.00000	43.25146	43.25260	43.25260
7	110	40.63783	40.63279	40.63439	0.00000	0.00000	0.00000	40.63316	40.63435	40.63436
7	120	38.26033	38.25503	38.25668	0.00000	0.00000	0.00000	38.25541	38.25663	38.25666
7	130	36.09311	36.08760	36.08928	0.00000	0.00000	0.00000	36.08798	36.08924	36.08927
7	140	34.11076	34.10509	34.10680	0.00000	0.00000	0.00000	34.10547	34.10675	34.10678
8	60	45.63658	45.65571	45.67313	45.67083	45.67630	45.64789	45.67962	45.68269	45.68268
8	70	39.02327	39.06153	39.08246	39.08357	39.08403	39.05511	39.08773	39.09204	39.09194
8	80	33.14427	33.18061	33.20407	33.21023	33.20378	33.17749	33.20755	33.21310	33.21311
8	90	27.99941	28.00525	6.42577	28.04268	28.02814	49.38166	28.03168	28.03834	28.03846
8	100	23.54710	23.50352	23.52908	23.54804	23.52537	23.51052	23.52848	23.53610	23.53631
8	110	19.72258	19.62733	19.65265	19.67728	19.64767	19.63984	19.65019	19.65853	19.65872
8	120	16.45462	16.31913	16.34356	16.37220	16.33765	16.33672	16.33953	16.34832	16.34860
8	130	13.67520	13.51728	13.54033	13.57101	13.53384	13.53918	13.53508	13.54407	13.54442
8	140	11.32274	11.16002	11.18137	11.21220	11.17461	11.18529	11.17525	11.18423	11.18446

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Table 6 – continued from previous page

Set	K	Merton-0	Merton-$\hat{t}$	Merton-Higher	Kou-0	Kou-$\hat{t}$	Kou-Higher	Gaussian	Integral	FFT
9	60	43.79142	43.81443	43.84460	43.84463	43.84215	43.80885	43.84320	43.84951	43.84949
9	70	36.53173	36.55193	36.58869	36.59579	36.58333	36.54780	36.58210	36.59256	36.59245
9	80	30.11082	30.08057	30.12140	30.13917	30.11291	30.07990	30.10864	30.12350	30.12358
9	90	24.55595	24.45411	24.49640	24.52395	24.48521	24.45808	24.47781	24.49653	24.49677
9	100	19.82714	19.66996	19.71142	19.74449	19.69840	19.67878	19.68837	19.70981	19.71017
9	110	15.85956	15.68074	15.71966	15.75341	15.70580	15.69381	15.69395	15.71678	15.71709
9	120	12.58009	12.40945	12.44482	12.47569	12.43099	12.42571	12.41816	12.44118	12.44161
9	130	9.90942	9.76410	9.79553	9.82175	9.78234	9.78232	9.76925	9.79152	9.79200
9	140	7.76384	7.64930	7.67680	7.69799	7.66460	7.66826	7.65181	7.67269	7.67301
10	60	55.40478	55.40389	55.40331	0.00000	0.00000	0.00000	55.40388	55.40432	55.40431
10	70	51.14487	51.14383	51.14433	0.00000	0.00000	0.00000	51.14382	51.14430	51.14424
10	80	47.35810	47.35692	47.35745	0.00000	0.00000	0.00000	47.35692	47.35742	47.35738
10	90	43.97575	43.97447	43.97501	0.00000	0.00000	0.00000	43.97446	43.97498	43.97497
10	100	40.94105	40.93968	40.94024	0.00000	0.00000	0.00000	40.93968	40.94020	40.94021
10	110	38.20691	38.20548	38.20604	0.00000	0.00000	0.00000	38.20547	38.20600	38.20602
10	120	35.73406	35.73257	35.73313	0.00000	0.00000	0.00000	35.73257	35.73310	35.73313
10	130	33.48955	33.48802	33.48858	0.00000	0.00000	0.00000	33.48802	33.48854	33.48859
10	140	31.44557	31.44401	31.44456	0.00000	0.00000	0.00000	31.44401	31.44453	31.44457
11	60	78.69712	78.69708	78.69710				78.69708	78.69710	77.85372
11	70	77.14730	77.14726	77.14728				77.14726	77.14728	76.65877
11	80	75.73489	75.73485	75.73486				75.73485	75.73486	75.43154
11	90	74.43584	74.43579	74.43581				74.43579	74.43581	74.23713
11	100	73.23221	73.23217	73.23219				73.23217	73.23219	73.09640
11	110	72.11024	72.11019	72.11021				72.11019	72.11021	72.01415
11	120	71.05903	71.05898	71.05900				71.05898	71.05900	70.98905
11	130	70.06987	70.06981	70.06983				70.06981	70.06983	70.01765
11	140	69.13558	69.13553	69.13555				69.13553	69.13555	69.09581
12	60	42.13739	42.33839	42.52323	42.22891	42.23686	42.19614	42.21561	42.23249	42.23245
12	70	33.88598	34.04443	34.24842	33.95846	33.94830	33.90020	33.87595	33.94073	33.94052
12	80	26.50817	26.45817	26.62575	26.44454	26.41143	26.37217	26.29998	26.40711	26.40725
12	90	20.09173	19.90351	20.03422	19.93497	19.90143	19.87370	19.79288	19.90139	19.90188
12	100	14.71954	14.54685	14.64079	14.58468	14.56256	14.54534	14.47973	14.56468	14.56537
12	110	10.47609	10.36842	10.43118	10.39900	10.38619	10.37712	10.32886	10.38888	10.38944
12	120	7.29432	7.23531	7.27675	7.25778	7.24994	7.24596	7.21039	7.25241	7.25313
12	130	4.99603	4.96285	4.99131	4.97928	4.97352	4.97211	4.94453	4.97550	4.97623
12	140	3.37934	3.35904	3.38006	3.37129	3.36614	3.36565	3.34271	3.36757	3.36798
13	60	43.27851	43.34283	43.38191	43.39178	43.39095	43.37332	43.39776	43.39600	43.39597
13	70	35.73319	35.78936	35.84290	35.85264	35.84440	35.82329	35.84821	35.85065	35.85051
13	80	29.07916	29.00799	29.07222	29.08611	29.06597	29.04465	29.06133	29.07288	29.07296
13	90	23.32998	23.10383	23.17381	23.19174	23.16120	23.14257	23.14491	23.16841	23.16870
13	100	18.42426	18.11521	18.18580	18.20360	18.16874	18.15454	18.14153	18.17594	18.17638
13	110	14.31557	14.01456	14.08129	14.09515	14.06201	14.05274	14.02747	14.06890	14.06927
13	120	10.96488	10.72387	10.78383	10.79258	10.76434	10.75957	10.72684	10.77067	10.77119
13	130	8.30718	8.13606	8.18810	8.19251	8.16974	8.16847	8.13283	8.17537	8.17594
13	140	6.24704	6.13435	6.17857	6.18003	6.16198	6.16302	6.12792	6.16684	6.16720
14	60	42.64420	42.80838	42.88868	42.84842	42.84956	42.85995	42.86585	42.84557	42.84553
14	70	34.80701	34.89491	35.01907	34.91794	34.92673	34.94175	34.93394	34.92310	34.92291
14	80	27.91328	27.71754	27.87157	27.73154	27.75133	27.76813	27.71872	27.74854	27.74864
14	90	21.91507	21.46945	21.63531	21.49342	21.51905	21.53440	21.43068	21.51633	21.51672
14	100	16.74749	16.26744	16.43058	16.30674	16.33023	16.34254	16.20079	16.32715	16.32772
14	110	12.48471	12.11025	12.25968	12.15665	12.17513	12.18421	12.03125	12.17188	12.17235
14	120	9.16514	8.89887	9.03103	8.94202	8.95721	8.96378	8.81755	8.95411	8.95474
14	130	6.67717	6.48464	6.60148	6.51743	6.53169	6.53686	6.40513	6.52901	6.52967
14	140	4.83904	4.70760	4.81217	4.72677	4.74100	4.74578	4.63201	4.73889	4.73928
15	60	42.46311	42.65503	42.74828	42.68804	42.69200	42.71938	42.71386	42.67936	42.67933
15	70	34.49097	34.61020	34.76641	34.59165	34.61874	34.66338	34.63753	34.60842	34.60819
15	80	27.45258	27.25782	27.45772	27.19536	27.25646	27.31063	27.22600	27.25110	27.25118
15	90	21.29960	20.81798	21.02896	20.76514	20.84195	20.89244	20.73243	20.83743	20.83783
15	100	15.94175	15.45619	15.65780	15.44629	15.50926	15.54504	15.34845	15.50289	15.50350
15	110	11.50968	11.20358	11.36928	11.22692	11.26673	11.29229	11.10447	11.25904	11.25956
15	120	8.11505	7.96315	8.08929	7.99679	8.02093	8.03422	7.88435	8.01325	8.01394
15	130	5.64974	5.57270	5.66519	5.60094	5.61869	5.62349	5.51079	5.61196	5.61268
15	140	3.90326	3.85511	3.92362	3.87109	3.88828	3.88853	3.80276	3.88289	3.88331

Table 7: Vanilla Call Option Values: $S_{t_0} = 100, r = 0.05, q = 0.02, \sigma_{\text{CGMY}} = 0.2, T = 0.5$

Set	K	Merton-0	Merton-$\hat{t}$	Merton-Higher	Kou-0	Kou-$\hat{t}$	Kou-Higher	Gaussian	Integral	FFT
1	60	40.49538	40.59943	40.56459	40.57386	40.76381	40.37188	40.59604	40.61719	40.61713
1	70	30.56892	31.34783	30.81433	30.97008	31.36401	30.10059	31.04294	31.10737	31.10683
1	80	21.45264	22.97711	26.06216	21.89413	22.30212	20.75596	21.82015	22.01232	22.01224
1	90	14.72190	14.51644	16.65821	14.30032	14.08509	13.18957	13.55025	13.94213	13.94312
1	100	8.45546	7.67883	8.42999	7.95176	7.69909	7.39768	7.40432	7.71201	7.71378
1	110	3.66404	3.63917	3.85791	3.72022	3.68137	3.63015	3.58279	3.70632	3.70761
1	120	1.53626	1.55277	1.63450	1.58614	1.56195	1.55776	1.51207	1.57232	1.57347
1	130	0.56965	0.63062	0.69420	0.61965	0.61070	0.59523	0.56014	0.60774	0.60847
1	140	0.17889	0.27075	0.33297	0.21516	0.23791	0.19511	0.19014	0.22314	0.22337
2	60	40.88493	41.05839	41.31669	40.89016	40.93764	40.74327	40.90933	40.92844	40.92840
2	70	31.59159	31.99909	32.29494	31.84320	31.90295	31.67737	31.86924	31.90505	31.90478
2	80	23.41541	23.64093	23.89992	23.63751	23.62642	23.41980	23.58948	23.64341	23.64357
2	90	16.83038	16.45279	16.65605	16.64653	16.50033	16.35333	16.46424	16.52948	16.53017
2	100	11.36153	10.77487	10.92088	11.01861	10.83031	10.75303	10.79807	10.86205	10.86310
2	110	7.00318	6.65562	6.75181	6.82385	6.69424	6.67025	6.66762	6.72063	6.72146
2	120	4.01957	3.90062	3.96178	3.99039	3.92363	3.92656	3.90270	3.94229	3.94327
2	130	2.21075	2.19090	2.23112	2.23546	2.20404	2.21314	2.18763	2.21595	2.21681
2	140	1.18227	1.19519	1.22365	1.21456	1.20175	1.20693	1.18865	1.20862	1.20902

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Table 7 – continued from previous page

Set	K	Merton-0	Merton-$\hat{t}$	Merton-Higher	Kou-0	Kou-$\hat{t}$	Kou-Higher	Gaussian	Integral	FFT
3	60	40.80614	41.01977	41.32001	40.81585	40.87432	40.64895	40.83710	40.85616	40.85611
3	70	31.38372	31.90212	32.30698	31.66967	31.75040	31.48006	31.70212	31.74113	31.74082
3	80	23.11673	23.40445	23.75485	23.34453	23.34207	23.09200	23.28504	23.34896	23.34911
3	90	16.50009	16.05080	16.31346	16.25224	16.08267	15.90705	16.02581	16.10717	16.10791
3	100	10.91865	10.28494	10.46413	10.54347	10.34088	10.25061	10.29224	10.37177	10.37291
3	110	6.49622	6.18238	6.29315	6.34396	6.22268	6.19456	6.18606	6.24888	6.24977
3	120	3.59523	3.51278	3.57934	3.59194	3.53642	3.53672	3.51051	3.55456	3.55557
3	130	1.91200	1.91078	1.95330	1.94771	1.92378	1.92864	1.90512	1.93487	1.93572
3	140	0.98988	1.01186	1.04196	1.02485	1.01737	1.01707	1.00336	1.02330	1.02369
4	60	40.58412			40.56909	40.60454	40.99690	40.58583	40.65368	40.65363
4	70	31.03219	31.56511	31.42442	31.18652	31.53264	30.92035	31.20156	31.32535	31.32501
4	80	22.57122	22.95198	23.55649	22.71975	22.79712	22.35455	22.56480	22.71042	22.71067
4	90	15.67129	15.37753	15.65866	15.53331	15.36987	15.16874	15.25316	15.37075	15.37168
4	100	10.04948	9.67656	9.82284	9.85816	9.70335	9.61832	9.64253	9.72469	9.72596
4	110	5.98307	5.76791	5.86800	5.89532	5.79122	5.74031	5.74798	5.81102	5.81193
4	120	3.37258	3.30965	3.41030	3.38444	3.32112	3.25645	3.27508	3.33256	3.33350
4	130	1.80781	1.88708	2.01213	1.87988	1.87016	1.77033	1.81063	1.86704	1.86780
4	140	0.93865	1.11944	1.28591	1.01439	1.06245	0.91915	0.98514	1.03875	1.03908
5	60	40.99521	41.08507	41.21646	41.04001	41.04912	40.96433	41.05988	41.06629	41.06625
5	70	31.77218	32.14200	32.31589	32.11311	32.13469	32.01730	32.14308	32.15820	32.15794
5	80	23.81549	23.96082	24.15192	24.02294	23.99782	23.87281	23.99505	24.02537	24.02553
5	90	17.44138	16.91935	17.10134	17.12473	16.99003	16.88866	16.96979	17.01993	17.02059
5	100	12.04579	11.31303	11.46966	11.56832	11.38708	11.32428	11.35102	11.41651	11.41751
5	110	7.65138	7.20370	7.32805	7.40059	7.26202	7.23239	7.21966	7.28745	7.28825
5	120	4.57561	4.41424	4.50992	4.53669	4.45518	4.44285	4.41655	4.47491	4.47584
5	130	2.64679	2.63994	2.71495	2.70615	2.66654	2.65676	2.63667	2.68062	2.68145
5	140	1.50888	1.56458	1.62578	1.59067	1.57917	1.56463	1.55867	1.58868	1.58908
6	60	41.47347	41.50173	41.55882	41.51189	41.51966	41.45514	41.52360	41.53300	41.53297
6	70	32.94456	33.04037	33.10842	33.07630	33.07563	33.00263	33.07755	33.09335	33.09322
6	80	25.49885	25.50368	25.57589	25.58113	25.54723	25.48024	25.54579	25.56817	25.56839
6	90	19.29660	19.09573	19.16597	19.21474	19.13832	19.08757	19.13302	19.16049	19.16102
6	100	14.21648	13.90071	13.96445	14.03612	13.93712	13.90549	13.92854	13.95843	13.95914
6	110	10.16079	9.87528	9.93015	9.99817	9.90439	9.88886	9.89392	9.92336	9.92392
6	120	7.06872	6.87925	6.92496	6.97506	6.90210	6.89689	6.89128	6.91807	6.91877
6	130	4.82297	4.72329	4.76081	4.79110	4.74139	4.74081	4.73144	4.75436	4.75506
6	140	3.25272	3.21245	3.24317	3.25742	3.22704	3.22695	3.21860	3.23732	3.23772
7	60	44.74990	44.74123	44.75286	36.49035	35.54825	35.53293	44.74537	44.75188	44.75187
7	70	37.98902	37.96894	37.98208	31.01758	30.39312	30.37934	37.97306	37.98092	37.98091
7	80	32.11752	32.08424	32.09831	26.27078	25.80998	25.79848	32.08816	32.09704	32.09717
7	90	27.08265	27.03776	27.05233	22.20287	21.82290	21.81374	27.04142	27.05097	27.05119
7	100	22.80456	22.75178	22.76652	18.74699	18.40514	18.39796	22.75517	22.76510	22.76538
7	110	19.19299	19.13664	19.15131	15.82888	15.50460	15.49887	19.13978	19.14987	19.15008
7	120	16.15764	16.10156	16.15026	13.37475	13.05939	13.02587	16.10449	16.11454	16.11484
7	130	13.61395	13.56099	13.57509	11.31581	11.00684	11.00242	13.56375	13.57363	13.57396
7	140	11.48585	11.43782	11.45151	9.59042	9.28823	9.28381	11.44045	11.45006	11.45027
8	60	41.11188	41.29939	41.53382	41.21439	41.22608	41.08627	41.24689	41.25636	41.25632
8	70	31.64231	32.41954	32.71502	32.34706	32.39022	32.20183	32.41462	32.43339	32.43310
8	80	23.73866	24.22086	24.53941	24.24996	24.26996	24.06338	24.29093	24.32335	24.32339
8	90	17.86121	17.03071	17.32744	17.30879	17.15693	16.98015	17.16701	17.21637	17.21688
8	100	12.63233	11.17141	11.41734	11.63616	11.32107	11.21305	11.31562	11.38028	11.38119
8	110	7.62129	6.80463	6.98627	7.18129	6.91720	6.88669	6.89800	6.96785	6.96868
8	120	3.98859	3.83905	3.95762	4.02323	3.90102	3.92657	3.87610	3.93781	3.93889
8	130	1.97819	2.00630	2.08063	2.08280	2.03544	2.08160	2.01117	2.05909	2.06010
8	140	0.94813	0.97806	1.02911	1.02057	0.99021	1.03067	0.96703	1.00458	1.00506
9	60	40.70454	40.98799	41.36666	40.75585	40.78982	40.58885	40.78289	40.79209	40.79204
9	70	30.98888	31.80854	32.47435	31.45534	31.54144	31.24622	31.51755	31.54288	31.54248
9	80	22.53227	23.10581	23.79647	22.86064	22.89357	22.55725	22.83009	22.89242	22.89246
9	90	16.06152	15.34187	15.87846	15.52497	15.30283	15.03624	15.19627	15.31667	15.31746
9	100	10.21915	9.22824	9.58975	9.57302	9.30152	9.15812	9.18136	9.33362	9.33497
9	110	5.43125	5.09259	5.31566	5.29078	5.15606	5.10369	5.05156	5.18612	5.18719
9	120	2.68721	2.63408	2.79420	2.73130	2.65498	2.62835	2.56329	2.67149	2.67260
9	130	1.22243	1.34622	1.50225	1.34100	1.31358	1.26791	1.22984	1.31266	1.31350
9	140	0.50711	0.72672	0.91152	0.62323	0.65092	0.57294	0.58356	0.63615	0.63647
10	60	44.06814	44.06399	44.07097	-6252.45562	48025.93009	47101.19934	44.06336	44.06846	44.06845
10	70	37.09507	37.08750	37.09432	1017.29092			37.08712	37.09230	37.09230
10	80	31.04217	31.03171	31.03811	-1590.30899	705.39436	672.97093	31.03148	31.03648	31.03664
10	90	25.86740	25.85494	25.86077	304.87056	-1718.49656	-1738.74654	25.85478	25.85946	25.85971
10	100	21.49358	21.48006	21.48528	466.50682	552.11719	555.75499	21.47996	21.48422	21.48453
10	110	17.82765	17.81392	17.81852	-848.64049	-89.18009	-89.92025	17.81385	17.81767	17.81791
10	120	14.77360	14.76029	14.76430	-508.02270	-7.67934	-7.71929	14.76024	14.76361	14.76394
10	130	12.24003	12.22757	12.23105	-456.40847	10.62208	10.66940	12.22753	12.23049	12.23085
10	140	10.14417	10.13279	10.13579	-323.87679	2.33371	2.34428	10.13276	10.13535	10.13558
11	60	57.86810	57.86764	57.86784	0.00000	0.00000	0.00000	57.86764	57.86783	57.86782
11	70	53.73960	53.73908	53.73929	0.00000	0.00000	0.00000	53.73907	53.73929	53.73923
11	80	50.07340	50.07282	50.07306	0.00000	0.00000	0.00000	50.07282	50.07305	50.07301
11	90	46.79767	46.79705	46.79729	0.00000	0.00000	0.00000	46.79704	46.79728	46.79727
11	100	43.85470	43.85404	43.85429	0.00000	0.00000	0.00000	43.85403	43.85428	43.85428
11	110	41.19763	41.19693	41.19719	0.00000	0.00000	0.00000	41.19693	41.19718	41.19719
11	120	38.78801	38.78729	38.78755	0.00000	0.00000	0.00000	38.78729	38.78754	38.78757
11	130	36.59401	36.59327	36.59353	0.00000	0.00000	0.00000	36.59327	36.59352	36.59356
11	140	34.58901	34.58826	34.58852	0.00000	0.00000	0.00000	34.58826	34.58851	34.58854
12	60	40.46260	40.54385	40.51177	40.52541	40.53888	40.37253	40.54152	40.53356	40.53350
12	70	30.64357	30.99517	30.70156	30.84375	30.91819	30.50410	30.88370	30.87664	30.87600
12	80	21.08989	22.32422	24.13700	21.45197	21.61423	20.77819	21.36314	21.49013	21.48997
12	90	13.44632	13.84417	17.25388	13.27342	13.19488	12.64207	12.45201	13.09009	13.09127
12	100	7.10171	6.76120	7.45497	6.80471	6.75390	6.63929	6.40582	6.75347	6.75557
12	110	2.97468	2.92119	3.10458	2.95135	2.93262	2.90774	2.80635	2.93613	2.93752
12	120	1.08162	1.21002	1.50578	1.15137	1.11614	1.06515	0.89170	1.10197	1.10306
12	130	0.22728	0.54002	0.63039	0.37350	0.41475	0.23740	0.23898	0.37549	0.37609

Continued on next page

Table 7 – continued from previous page

Set	K	Merton-0	Merton-$\hat{t}$	Merton-Higher	Kou-0	Kou-$\hat{t}$	Kou-Higher	Gaussian	Integral	FFT
12	140	0.02507	0.12296	-0.06527	0.10650	0.15971	-0.01256	0.08781	0.12548	0.12563
13	60	40.56597	40.87540	41.18584	40.73374	40.72997	40.69348	40.75910	40.74929	40.74924
13	70	30.82108	31.60753	32.17776	31.35251	31.36623	31.28315	31.41522	31.39432	31.39383
13	80	22.22861	22.80210	23.63372	22.49836	22.49636	22.34529	22.51804	22.50943	22.50935
13	90	15.62680	14.79658	15.62004	14.75860	14.61666	14.44798	14.49362	14.61347	14.61434
13	100	9.39971	8.35748	8.93894	8.56021	8.41528	8.31374	8.15010	8.42635	8.42798
13	110	4.46585	4.23768	4.58502	4.37432	4.31742	4.27743	4.09185	4.33222	4.33344
13	120	2.06212	2.04891	2.29628	2.09120	2.04335	2.01648	1.88843	2.04854	2.04968
13	130	0.78900	1.03170	1.28647	0.95032	0.94138	0.90031	0.85900	0.93719	0.93794
13	140	0.27055	0.55645	0.79688	0.42467	0.44144	0.39850	0.41458	0.43779	0.43804
14	60	40.45530	40.82603	41.23054	40.67913	40.68108	40.68936	40.70226	40.67187	40.67182
14	70	30.75659	31.49975	32.40004	31.18315	31.18487	31.20476	31.24401	31.16492	31.16432
14	80	21.80348	22.59232	24.32640	21.97668	21.98810	22.06480	22.06957	21.98177	21.98160
14	90	14.85360	14.34569	16.38080	13.64521	13.72977	13.87573	13.47524	13.75113	13.75220
14	100	8.31765	7.45857	8.59424	7.39562	7.45298	7.53325	6.74361	7.45235	7.45430
14	110	3.98840	3.56345	4.34886	3.49531	3.54567	3.59500	2.97664	3.54408	3.54543
14	120	1.67787	1.93356	3.13709	1.51164	1.55277	1.63440	1.27355	1.56693	1.56802
14	130	0.42153	1.14987	2.38245	0.70525	0.69915	0.76612	0.66913	0.70750	0.70813
14	140	0.04867	0.71882	1.62870	0.37114	0.36003	0.39128	0.39474	0.35610	0.35628
15	60	40.45460	40.80642	41.22056	40.67062			40.69268	40.65262	40.65257
15	70	30.76641	31.45989	32.43688	31.14740	31.15838	31.20216	31.20912	31.10347	31.10284
15	80	21.68349	22.52354	24.63609	21.81018	21.85884	22.01463	21.96428	21.83058	21.83036
15	90	14.54716	14.23723	17.06597	13.16433	13.39743	13.86884	13.17661	13.47334	13.47441
15	100	7.88695	7.11795	8.48602	6.93473	7.07152	7.29194	6.27480	7.07502	7.07705
15	110	3.19498	3.08974	3.59568	3.10786	3.16401	3.23703	2.76702	3.14892	3.15032
15	120	1.29717	1.37918	1.95122	1.07075	1.21113	1.29814	0.92249	1.22537	1.22650
15	130	0.30796	0.70065	1.43753	0.42109	0.42479	0.53673	0.35707	0.44626	0.44689
15	140	0.03145	0.38147	0.85693	0.19367	0.16541	0.20595	0.17399	0.16873	0.16890

K	Merton$-\hat{\mathrm{t}}$	Kou$-\hat{\mathrm{t}}$	Gaussian	Integral	FFT	Carr/Madan
10	90.14890	−2626.36238	90.14890	90.14890	90.14884	90.15710
20	80.29921	80.38055	80.29897	80.29900	80.29893	80.32790
30	70.46649	70.45811	70.46088	70.46119	70.46092	70.51780
40	60.68652	60.67086	60.67513	60.67649	60.67611	60.73970
50	51.03891	51.03127	51.03827	51.04220	51.04167	51.04820
60	41.69384	41.71241	41.72215	41.73070	41.73067	41.61040
70	32.91066	32.96102	32.97227	32.98735	32.98716	32.73030
80	24.99257	25.06486	25.07546	25.09790	25.09808	24.76880
90	18.21377	18.29067	18.29827	18.32707	18.32764	18.01840
100	12.74241	12.81007	12.81327	12.84556	12.84640	12.61910
110	8.59185	8.64534	8.64450	8.67657	8.67725	8.54280
120	5.62527	5.66622	5.66311	5.69188	5.69272	5.62780
130	3.61059	3.64238	3.63896	3.66277	3.66357	3.63870
140	2.29377	2.31908	2.31649	2.33504	2.33546	2.32950
150	1.45395	1.47432	1.47290	1.48672	1.48753	1.48790
160	0.92512	0.94134	0.94095	0.95094	0.95149	0.95370
170	0.59347	0.60596	0.60630	0.61340	0.61381	0.61590
180	0.38510	0.39419	0.39495	0.39995	0.40007	0.40180
190	0.25342	0.25947	0.26042	0.26395	0.26399	0.26520
200	0.16945	0.17293	0.17392	0.17641	0.17644	0.17720

Table 8: Carr & Madan Comparison: $C_{\text{up}}, C_{\text{down}} = 2, M = 5, G = 10, Y_{\text{up}}, Y_{\text{down}} = 0.5.$

$S_{t_0} = 100, r = 0.03, q = 0, \sigma_{\text{CGMY}} = 0, T = 0.5.$

$\mathbf{S_{t_0}}$	**Kou**$-\hat{\mathrm{t}}$	**Gaussian**	**Integral**	**Sepp**
90	4.35986	4.35986	4.35983	4.35990
100	9.22701	9.22701	9.22701	9.22700
110	15.96130	15.96130	15.96130	15.96130

Table 9: Sepp Comparison: $\lambda = 0, \eta_1 = 10, \eta_2 = 10, p = 0.5.$

$K = 100, r = 0.05, q = 0.02, \sigma_{\mathrm{Kou}} = 0.2, T = 1.$

$\mathbf{S_{t_0}}$	**Merton**$-\hat{\mathrm{t}}$	**Kou**$-\hat{\mathrm{t}}$	**Gaussian**	**Integral**	**Sepp**
90	8.16943	8.20488	8.18033	8.20489	8.20490
100	13.31579	13.35051	13.31847	13.35052	13.35050
110	19.75254	19.78597	19.75597	19.78597	19.78600

Table 10: Sepp Comparison: $\lambda = 3, \eta_1 = 10, \eta_2 = 10, p = 0.5.$

$K = 100, r = 0.05, q = 0.02, \sigma_{\mathrm{Kou}} = 0.2, T = 1.$

$\mathbf{S_{t_0}}$	**Merton**$-\hat{\mathrm{t}}$	**Kou**$-\hat{\mathrm{t}}$	**Gaussian**	**Integral**	**Sepp**
90	10.21568	10.24608	10.23856	10.24780	10.24780
100	15.51381	15.54394	15.53377	15.54617	15.54620
110	21.89490	21.92395	21.91412	21.92674	21.92670

Table 11: Sepp Comparison: $\lambda = 5, \eta_1 = 10, \eta_2 = 10, p = 0.5.$

$K = 100, r = 0.05, q = 0.02, \sigma_{\mathrm{Kou}} = 0.2, T = 1.$

K	T	Merton$-\hat{t}$	Kou$-\hat{t}$	Gaussian	Integral	FFT	Rogers/Zane
$\exp(-0.05)$	5	0.054319	0.050942	0.054645	0.054700	0.054701	0.054600
$\exp(-0.05)$	2	0.043611	0.044515	0.044559	0.044851	0.044854	0.046600
$\exp(-0.05)$	1	0.030930	0.031280	0.030979	0.031874	0.031896	0.031000
$\exp(-0.05)$	0.5	0.021937	0.019459	0.017936	0.019929	0.019948	0.017900
$\exp(-0.05)$	0.25	0.020205	0.012511	0.008406	0.010976	0.011351	0.008400
1	5	0.067166	0.064047	0.067515	0.067534	0.067526	0.067500
1	2	0.059433	0.060401	0.060428	0.060629	0.060632	0.006040
1	1	0.046673	0.047520	0.046897	0.047770	0.047726	0.046800
1	0.5	0.034320	0.034229	0.031090	0.033918	0.033762	0.030900
1	0.25	0.024553	0.023448	0.014748	0.018438	0.021721	0.014500
$\exp(0.05)$	5	0.082574	0.079724	0.082936	0.082809	0.082811	0.082800
$\exp(0.05)$	2	0.111296	0.111138	0.104836	0.081064	0.081064	0.080800
$\exp(0.05)$	1	0.070462	0.071545	0.070784	0.071447	0.071421	0.070400
$\exp(0.05)$	0.5	0.062495	0.061827	0.059267	0.062069	0.062094	0.059200
$\exp(0.05)$	0.25	0.064469	0.057556	0.051882	0.056939	0.056434	0.051900

Table 12: Rogers & Zane Comparison: $C_{\text{up}}, C_{\text{down}} = 1, G = M = 7.071068, Y_{\text{up}}, Y_{\text{down}} = 0.$

$S_{t_0} = 1, r = 0.05, q = 0, \sigma_{\text{CGMY}} = 0.$

K	T	GME	BS	Carr/Madan GME
60.65	0.5	40.26000	40.26000	40.25690
81.87	0.5	20.20766	20.20766	20.20500
90.48	0.5	13.45126	13.45126	13.44860
110.52	1	7.08109	7.08109	7.08205
122.14	1	4.00481	4.00481	4.00475
174.87	1	0.20056	0.20056	0.36594
271.82	1	0.00049	0.00049	0.00049

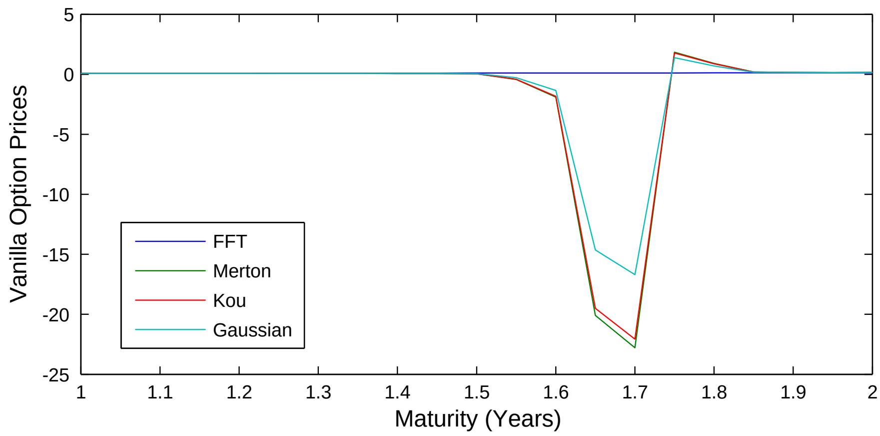
Table 13: Carr & Madan Comparison: $S_{t_0} = 100, r = 0.03, q = 0, \sigma = 0.25.$

Set	K	GME	MME	Integral	FFT
1	60	42.13184	42.58585	42.14525	42.14521
1	70	32.95650	33.63741	33.05063	33.05031
1	80	24.51777	25.28649	24.65855	24.65868
1	90	17.25947	17.86288	17.39069	17.39137
1	100	11.48019	11.71029	11.57308	11.57412
1	110	7.22857	7.08573	7.28809	7.28892
1	120	4.32846	4.02725	4.37054	4.37151
1	130	2.48097	2.23763	2.51624	2.51712
1	140	1.37147	1.23786	1.40316	1.40358
7	60	50.70991	50.64047	50.78860	50.78859
7	70	45.18795	45.06467	45.26325	45.26321
7	80	40.34447	40.18704	40.41348	40.41351
7	90	36.09698	35.92211	36.15856	36.15863
7	100	32.36908	32.18914	32.42309	32.42319
7	110	29.09240	28.91573	29.13928	29.13937
7	120	26.20697	26.03871	26.24741	26.24754
7	130	23.66073	23.50375	23.69552	23.69568
7	140	21.40880	21.26437	21.43871	21.43882
10	60	49.61539	49.49963	49.65147	49.65146
10	70	43.85913	43.6504	43.89737	43.89734
10	80	38.81904	38.5523	38.85678	38.85682
10	90	34.41361	34.11909	34.44924	34.44933
10	100	30.56416	30.26414	30.59685	30.59697
10	110	27.19856	26.90769	27.228	27.2281
10	120	24.25256	23.97948	24.27873	24.27888
10	130	21.66984	21.41891	21.69291	21.69309
10	140	19.40155	19.1743	19.42177	19.4219

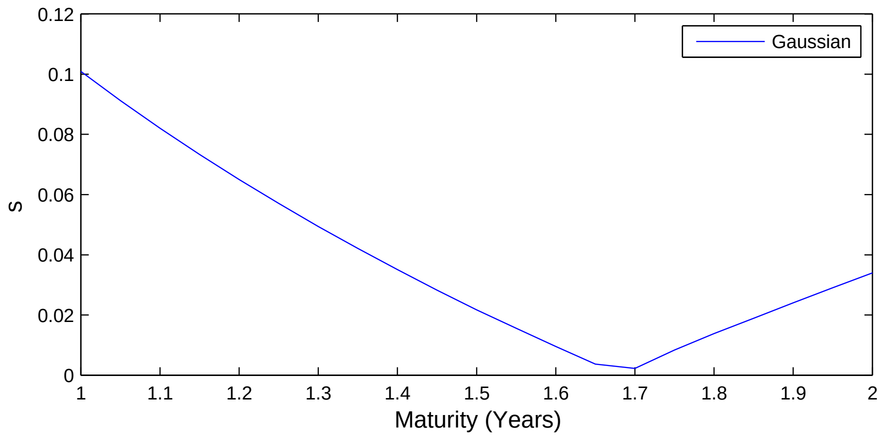
Table 14: GME, MME Comparison: $S_{t_0} = 100, r = 0.03, q = 0, \sigma_{\text{CGMY}} = 0.2, T = 1$.

Appendix C : Rogers & Zane's Graphs

- Results for $S_T = 1, K = \exp(0.05), r = 0.05, q = 0, \sigma_{\text{CGMY}} = 0$.



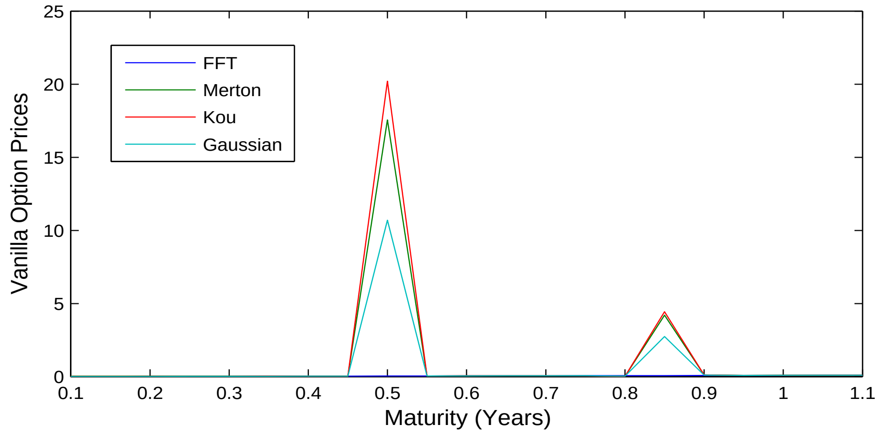
(a) Saddlepoint Option Values



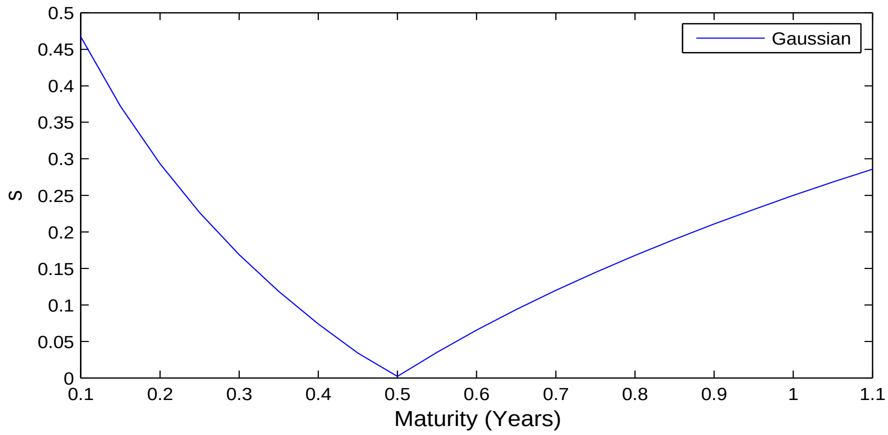
(b) Plot of s: Formula (3.1)

Figure 4: Saddlepoint Values for Rogers & Zane Comparison: $C_{\text{up}}, C_{\text{down}} = 1$, $G, M = 7.071068$.

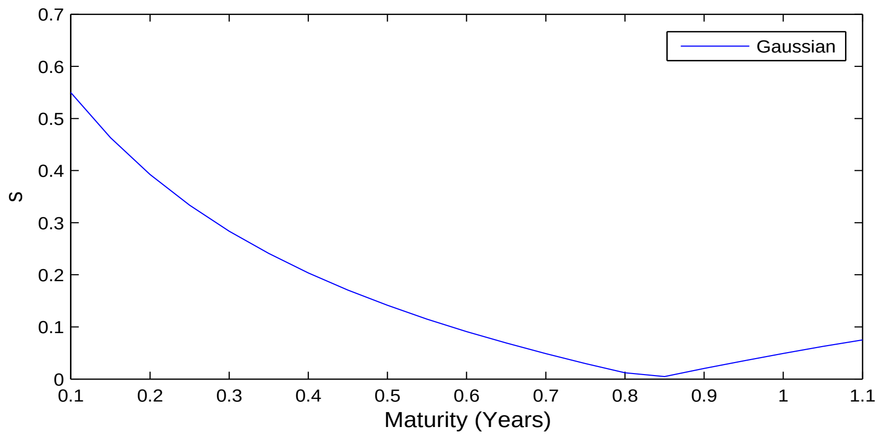
- Results for $S_T = 1, K = \exp(0.05), r = 0.1, q = 0.02, \sigma_{\mathrm{CGMY}} = 0$.



(a) Saddlepoint Option Values



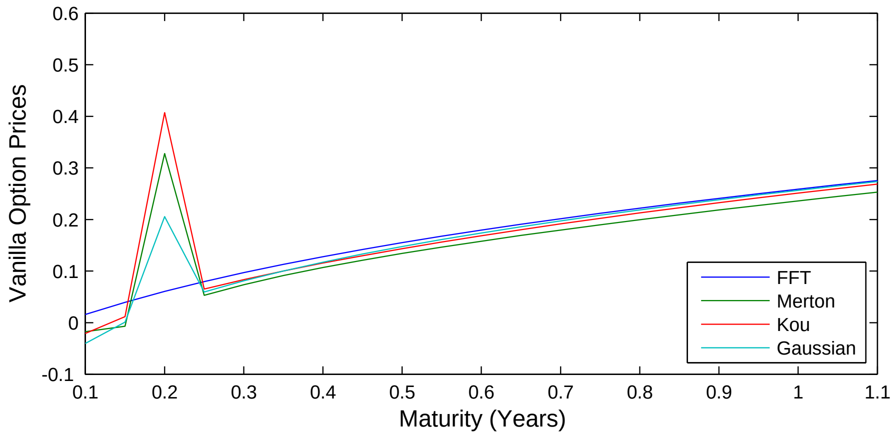
(b) Plot of s: Formula (3.1)



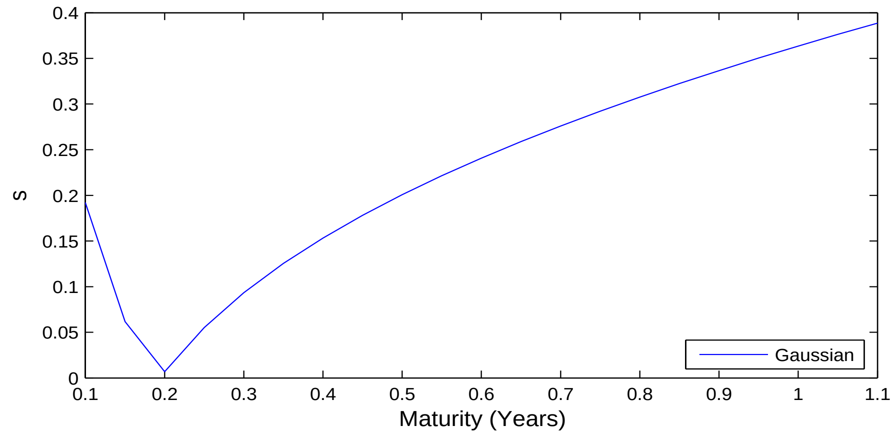
(c) Plot of s: Formula (3.1)

Figure 5: Saddlepoint Values for Rogers & Zane Comparison: $C_{\mathrm{up}}, C_{\mathrm{down}} = 1$, $G, M = 7.071068$.

- Results for $S_T = 1, K = \exp(0.05), r = 0.05, q = 0, \sigma_{\text{CGMY}} = 0$.

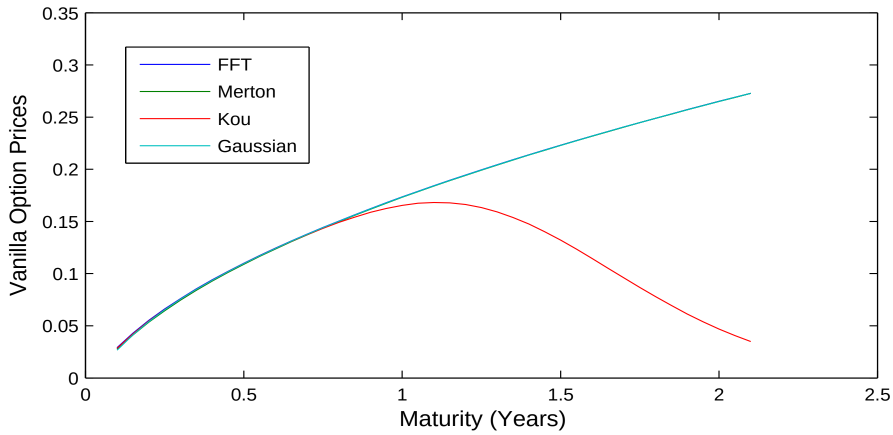


(a) Saddlepoint Option Values

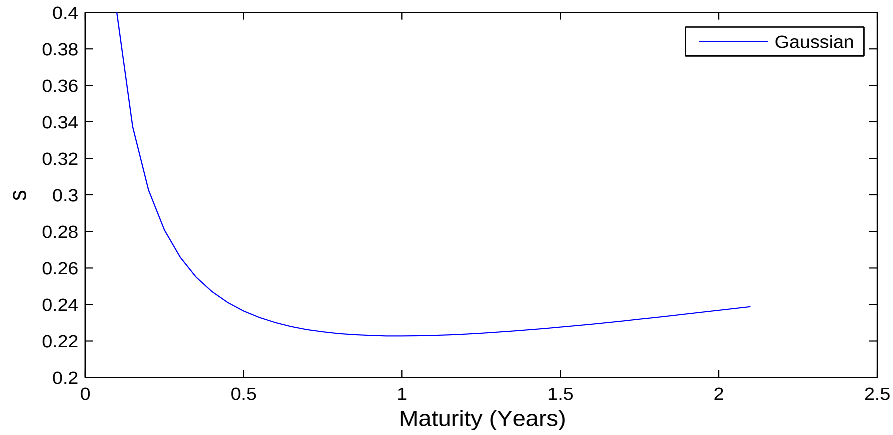


(b) Plot of s: Formula (3.1)

Figure 6: Saddlepoint Values for Rogers & Zane Comparison: $C_{\text{up}}, C_{\text{down}} = 0.833333$, $G = 0.816660, M = 40.816660$.

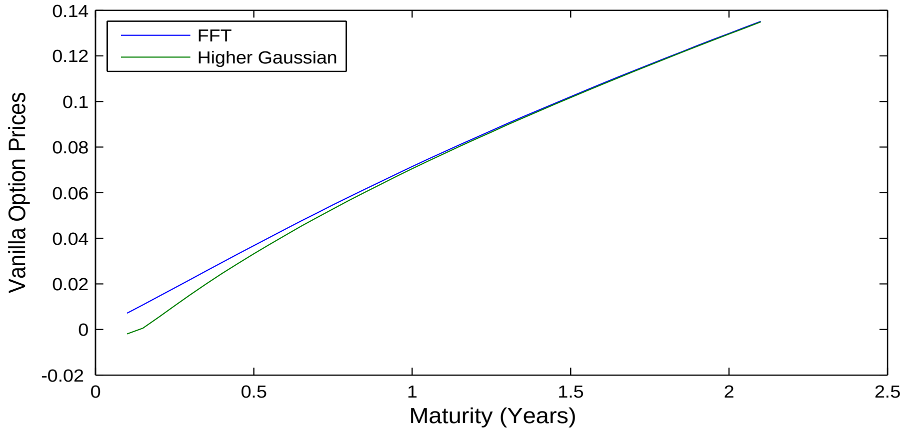


(a) Saddlepoint Option Values



(b) Plot of s: Formula (3.1)

Figure 7: Saddlepoint Values for Rogers & Zane Comparison: $C_{\text{up}}, C_{\text{down}} = 2, G = 5$, $M = 10$,$\text{Y}_{\text{up}}, Y_{\text{down}} = 0.5$.



(a) Saddlepoint Option Values

Figure 8: Saddlepoint Values for Rogers & Zane Comparison: $C_{\text{up}}, C_{\text{down}} = 1$, $G, M = 7.071068$.

Appendix D : Kou Model's Numerical Instability

We now investigate into why the Kou base distribution produces unreliable results for certain parameter sets. In order to facilitate this analysis, we consider the Saddlepoint approximation results obtained using the Kou model for two sets of CGMY parameters: set 1, where the Kou model produces relatively good results, and set 11, where the Kou model breaks down. The table below shows the BCON option price (computed using the original moment matching method) and the corresponding cdf and pdf computed for these two CGMY parameter sets, where $S_{t_0} = 1, K = 1, r = 0.05, q = 0.02, \sigma_{\text{CGMY}} = 0.2, T = 1$:

Set	**cdf**	**pdf**	**BCON Option Price**
1	0.481401	1.614352	0.493309
11	1	$6.22e-67$	$1.13e-71$

Table 15: Comparison Between BCON Option Prices For Parameter Sets 1 and 11

The differences between the inputs for the Kou cdf for different parameter sets, are the value that the cdf is being evaluated at, and the Kou parameters. For sets 1 and 11, this information is given in the table below:

Set	$\mathbf{y}_{\text{base}}$	λ	$\eta_{\mathbf{1}}$	$\eta_{\mathbf{2}}$	**p**	$\sigma_{\mathbf{Kou}}$
1	$-3.32E-06$	0.993399	17.634409	5.935484	0.556920	0.211717
11	$-3.22E-05$	239.775759	26.666667	13.333333	0.738796	1.242383

Table 16: Comparison Between Kou's Parameter for Parameter Sets 1 and 11

One can see that the Kou parameters for parameter set 11, for which the erroneous results are produced, are much more larger than that of parameter set 1. This indicates that the underlying problem originates from the values of the Kou parameters that have been obtained from either of the parameter attaining methods. However, it is necessary to confirm this.

There are several individual components which are used to calculate the final complementary cdf for the Kou model (equation (2.1)). However, by delving into the different components of the cdf function, one can see that the problem of the erroneous results originates with the values being computed from the Hh functions. As the Hh function is a recursive function which starts from computing the standard Gaussian pdf and cdf evaluated at a certain value, this suggests that the error originates from the initial value that is input into the function. The Hh function is used twice in calculating the Kou cdf, and the table below displays the initial values entered into the Hh function (for both runs of the function), and the corresponding results from calculating the standard Gaussian pdf and cdf functions evaluated at these values: (Note that when we calculate the cdf, we evaluate the standard Gaussian distribution at the negative of the input value).

Set	**Initial Value**	**pdf**	**cdf**
1	3.91203	$1.90E-04$	$4.58E-05$
1	1.07812	0.223106	0.140490
11	30.49502	$4.63E-203$	$1.52E-204$
11	19.20032	$3.54E-81$	$1.84E-82$

Table 17: Comparison Between Standard Gaussian pdf and cdf for Kou Parameters

One can see that for set 11, since the cdf and pdf values are very small, this would impact on the final output from the recursive Hh function. Therefore, our intuition was correct in that the Kou parameters attained from the moment matching method were causing the Kou cdf function to produce unreliable results.

The Kou parameters are based on calculating the ratio of the up components and ratio of the down components for the third and fourth derivatives of the CGMY cumulant function (see Section 3.2). As this ratio is larger for set 11 than for set 1, this would in turn produce Kou parameters which are larger. Since the method of obtaining the Kou parameters didn't involve selecting a pre-determined Kou parameter value (as it did for the Merton model), and Kou's cumulant generating function naturally splits into up and down components, we can't alter the Kou or CGMY values that determine the Kou parameters.

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